Economical evaluation of vehicles fleets' replacement considering potential environmental impacts resulting from carbon emissions

Miguel José Ferreira da Silva

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Supervisors: Prof. João Manuel Marcelino Dias Zambujal de Oliveira
Prof. Maria Isabel Craveiro Pedro

Examination Committee

Chairperson: Prof. Francisco Miguel Garcia Gonçalves de Lima
Supervisor: Prof. Maria Isabel Craveiro Pedro
Member of the committee: Prof. Maria Margarida Martelo Catalão Lopes de Oliveira Pires Pina

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Resumo

Hoje em dia as empresas necessitam de ter estruturas de custos eficientes para ser competitivas, por essa razão, a utilização de modelos económicos para analisar processos de substituição é muito importante e decisiva. A aquisição de veículos com emissões de CO_2 mais reduzidas é estimulada pelos governos que usam o enquadramento legal para aumentar os impostos associados a veículos com emissões de CO_2 mais elevadas.

A contribuição para este estudo é a definição de metodologias de fluxos de caixa descontados que utilizam ambientes determinísticos e estocásticos para avaliar o processo de substituição considerando os fluxos de caixa associados aos veículos e o enquadramento legal associado aos impostos e empresas de rent-a-car. As variáveis mais relevantes para o processo de substituição são o ISV (imposto sobre veículos), valor do investimento, o valor residual e os custos de manutenção. A incerteza associada aos custos de manutenção foi considerada na análise estocástica. Outro *input* importante para as metodologias foi a restrição legal de cinco anos que considera que as empresas de rent-a-car estão obrigadas a ter veículos que não ultrapassem os cinco anos. Isto afeta a maturidade do veículo e é importante para o processo de substituição. Considerando a substituição de veículos a gasolina e diesel por veículos híbridos existe a tendência para substituições rápidas e para substituir os veículos no primeiro período. As emissões de CO_2 relacionadas com o enquadramento legal e os impostos associados afetam os níveis de substituição e são relevantes para o processo de substituição.

Palavras-chave: processo de substituição, veículos, emissões de CO_2, enquadramento legal, determinístico, estocástico.

Abstract

Nowadays the companies need to have efficient cost structures to be competitive, for that reason, the use of economic models to analyze the replacement process is very important and decisive. The acquisition of vehicles with lower CO_2 emissions is stimulated by governments that use legal frameworks to increase taxes related to the vehicles that have higher CO_2 emissions.

The contribution of this study is the definition of discounted cash-flows methodologies that use deterministic and stochastic environments to evaluate the replacement process considering the cash-flows related to vehicles and the legal framework related to taxes and rent-a-car companies. The most relevant variables to the replacement process are the ISV (vehicles tax), investment value, residual value and the maintenance costs. The uncertainty related to the maintenance costs is considered for the stochastic environment. Other important input for the methodologies is the legal constraint of five years that says that the rent-a-car companies are obliged to have vehicles with not more than five years. This affects the maturity of the vehicles and is important to the replacement process. Considering the replacement of gasoline and diesel vehicles by hybrid vehicles there is the tendency to have fast vehicle`s replacement and to choose to replace the vehicles in the first period. The CO_2 emissions related to the CO_2 legal framework and CO_2 taxes affect the replacement levels and are relevant to the replacement process.

Keywords: replacement process, vehicles, CO_2 emissions, legal framework, deterministic, stochastic.

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Abbreviation List

AD – Accumulated Depreciation
BV – Book Value
DC – Depreciation Cost
EAC – Equivalent Annual Cost
FCFF – Free Cash Flows to the Firm
I – Investment Value
INS – Insurance Cost
INSP – Inspection Cost
ISV – Vehicles Tax
IUC – Circulation Tax
RO – Real Options
RPA – Replicating Portfolio Approach
RNA – Risk Neutral Approach
RV – Residual Value

SV – Salvage Value

OC – Opportunity Cost

1 Introduction

1.1 Background

In the today's world in which the markets show high levels of uncertainty and trough efficient cost structures it is essential that the companies create sustainable competitive advantages. Therefore, it is also fundamental that the selling and buying decisions of vehicles should be based on economic analysis under uncertainty environments, similar to the ones that exist on the markets.

Nowadays, the search for sustainable economic solutions is stimulated by governmental organizations through a higher tax burden on products that use the environmental resources in a non-renewable way. Besides the financial aspect, initiatives such as, the Green Procurement Program in the European Union and the existence of Ecolabels that promote the products and equipment that operate in a sustainable way have been causing a change of paradigm in the corporate procurement. In the same way, companies must foment economic appraisals for the asset replacement problem that integrate environmental variables to minimize costs and maximize the sustainability of those replacement operations (ECE, 2012).

The sector of Light Duty Vehicles represents 10% of CO_2 in the whole world (EESC, 2010). This evidence provides enough motivation for analyzing the environmental impacts of gas emissions (especially carbon emissions) on the asset replacement process. In 1997, an agreement was reached by 159 nations, where several goals were defined to promote the reduction of pollution gases emissions. This agreement was known as the Kyoto Protocol and it was created under the United Nations Convention on Climate Change. The objective of this Protocol is to establish compulsory goals for emissions reduction in 37 industrialized countries. They should, in average, reduce 5% below the 1990' levels, for a 5 years period, between 2008 and 2012. In order to fulfill the agreement, some countries defined additional measures where the goal was to reduce the carbon emissions on 25%, concerning 2006' values (EESC, 2010). This new goal also implied more severe penalties to old vehicles that showed high levels of CO_2 emissions.

In Portugal 153 342 new light passenger vehicles were bought in 2011, and approximately 20% of those cars were bought by rent-a-car companies [(1)]. These data reveals that Portuguese renting industry is responsible for considerable CO_2 emissions and that is relevant to analyze the effect of CO_2 emissions on vehicles` replacement from rent-a-car firm point of view.

[1] ARAC (Portuguese rent-a-car association); ACAP (Portuguese vehicles association) - Annex 1

1.2 Relevance and motivation

The term sustainability implies a response to the current population needs without threatening the resources availability for future generations. Therefore, natural resources should be used economically and efficiently, without harming the environment`s future.

Some studies developed in air transportation sectors and increasing automobiles` $C0_2$ emissions demonstrate the need for environmental valuations for vehicles. There is evidence that different technologies create diverse environmental impacts and consequently different replacement levels. It seems existing a relationship between environmental and technological variables that impacts replacement policy (Hynes, 2005). For this reason it`s important to evaluate environmental costs and study their impact on the economic appraisal.

The first main motivation of the author is to develop an evaluation model adapted to economic and sustainable principles and practices. Another motivation is trying to elaborate a simple and explicit model that can be understood by the companies and by the decisions makers.

The last motivation is to create a model adapted to real economic conditions. This means that uncertainty related to the input parameters and also legal framework related to rent-a-car vehicles must be considered.

Therefore the analysis should be focus on the environmental costs and taxes related to vehicles and on its impact in the replacement cost and level. This document will try to answer the question: are the environmental taxes leading to a more environmental concerned replacement process? Thus the challenge is to analyze economic and environmental evaluation of vehicles` replacement. In other words, to analyze the selling and buying decisions of the vehicles from rent-a-car firm point of view and considering legal framework related to CO_2 emissions.

1.3 Dissertation objectives and framework

The dissertation will pursue the following objectives:

1. Characterize and analyze vehicles historical data.
2. Analyze methodologies used in economical replacement evaluation.
3. Analyze economical and environmental criteria and parameters.
4. Implement a model in order to analyze replacement timing and level in deterministic environments, considering the presence and the absence of environmental costs.
5. Implement a model in order to analyze replacement timing and level in uncertain environments, considering the presence and the absence of environmental costs.

To develop this dissertation the following framework will be structure in seven steps. Step one will define the case study and the problem and step two will define the Portuguese legal framework related to rent-a-car vehicles. Step three will show the most used solutions presented in academic books and papers. Step number four will determine some assumptions and simplifications and also suitable models to apply to the case study and to the vehicle`s replacement process. The data used in this analysis will also be presented in step four. The fifth step is to perform an historical analysis of vehicles data. The historical analysis` results will be used to fill the parameters values needed to the methodologies. Step number five will present the results from the calculation of the replacement timing and replacement level according to the environments (deterministic or uncertain) with and without the legal framework related to CO_2 emissions. Step number six will verify the model robustness and developed a sensitivity analysis. Step number seven will present the conclusions about degree of achievement of the objectives and the project conclusions.

1.4 Case study definition

To study the replacement process it was created a case study that is an academic example about vehicles` replacement. There were defined three vehicles: a gasoline vehicle, an hybrid vehicle (gasoline + battery energy) and a diesel vehicle. The objective is to analyze replacement timing and level and also the impact of CO_2 emissions (environmental legal framework) in the replacement. In order to maintain similar vehicles performance and to create feasible/realistic replacements there were defined the following vehicles and the following criteria (Table 1).

Table 1 - Engine size, Maximum power and Maximum speed for case study vehicles

Models	**Engine size (cm^3)**	**Maximum power (cv/rpm)**	**Maximum speed (km/h)**
(1) Toyota Auris 1.6 Exclusive	1598 (gasoline)	124/6000	190
(2) Toyota Auris 1.8 HSD	1798 (hybrid)	99/5200	180
(3) Toyota Auris 2.0 Exclusive	1998 (diesel)	126/3600	195

First, it is important to understand that the objective was not to create a replacement of identical vehicles but an example of feasible/realistic replacements.

For that reason this analysis considered three vehicles with the same brand (Toyota) and model (Auris). The main difference is the type of fuel. Toyota Auris 1.6 is the gasoline vehicle for Toyota Auris model, Toyota Auris 1.8 HSD is the hybrid vehicle for Toyota Auris model (that also uses gasoline) and Toyota Auris 2.0 is the diesel vehicle for Toyota Auris model.

This study typified two vehicles` configurations, which were associated to two virtual companies – Portuguese rent-a-car companies. Table 2 shows the type of vehicles for each virtual company. These vehicles were bought in the period 0 and a new replacement can only occur after one year of utilization (period 1).

Table 2 - Virtual companies and their vehicles

Virtual Company	Models
A	Toyota Auris 1.6
B	Toyota Auris 2.0

In order to reduce CO_2 emissions the companies decided that the next replacement should only consider hybrid vehicles. The vehicle considered for replacement is Toyota Auris 1.8 as you can see in table 3.

Table 3 - Virtual companies and their new vehicles

Virtual Company	Challenger type
A	Toyota Auris 1.8
B	Toyota Auris 1.8

For their activity, the rent-a-car companies need to know the timing of replacement for each configuration and to understand the effect of CO_2 emissions legal framework in the replacement process. Table 4 presents the $C0_2$ emissions and the Average Fuel consumption of the case study vehicles**.**

Table 4 - $C0_2$ emissions and Average Fuel consumption of the vehicles *with 0.0015 g/km

	$C0_2$ emissions (g/km)	**Average Fuel consumption (l/100km)**
Toyota Auris 1.6	153	7.1
Toyota Auris 1.8	89	3.8
Toyota Auris 2.0	138*	5.2

2 Legal framework

This topic will explain the Portuguese legal framework related to rent-a-car companies and to rent-a-car vehicles. There are three types of taxes related to rent-a-car vehicles: ISV ("Imposto sobre veículos" – Vehicles tax), IUC ("Imposto único de circulação" – Circulation tax) and VAT ("Imposto de valor acrescentado" – Value added tax) - (CISV - Decreto de Lei nº 82-D/2014, 2015; CIUC - Decreto de Lei nº 82-B/2014, 2015).

2.1 Vehicles tax (ISV)

Vehicles tax (ISV) is a tax paid only one time and after the vehicle registration. It evaluates two elements: $C0_2$ emissions (environmental impact) and engine size. The following tables demonstrate the tax calculation for different values of $C0_2$ emissions and engine size. The following data is related to light passengers' vehicles. Table 5 presents ISV (Engine size) - (CISV - Decreto de Lei nº 82-D/2014, 2015; CIUC - Decreto de Lei nº 82-B/2014, 2015).

Table 5 - ISV (Engine size)

Engine size category (cm3)	Rate per engine size (€/cm3)	Correction factor (€)
Up to 1250	1,00	740,50
More than 1250	4,70	5362.67

The ISV (Engine Size) is calculated using equation 1:

$$ISV\left(Es\right) = Es \times RtEs - CfEs \quad (1)$$

Es: Engine size; RtEs: Rate per engine size; CfEs: Correction factor of Engine size

Table 6 present ISV ($C0_2$ emissions) values for gasoline vehicles. (CISV - Decreto de Lei nº 82-D/2014, 2015)

Table 6 - ISV ($C0_2$) for gasoline vehicles

$C0_2$ emissions category (g/Km)	Rate per $C0_2$ emissions (€.Km/g)	Correction factor (€)
Up to 115	4.15	390.35
116 to 145	37,91	4281.66
146 to 175	44.00	5161.20
176 to 195	111.85	17047.04
More than 195	147.69	24021.60

The ISV (CO_2 emissions) is calculated using the following equation:

$$ISV\left(C0_2\,e\right) = CO_2e \times RtCO_2e - Cf\;CO_2\;e \qquad (2)$$

CO_2e: $C0_2$emissions; $RtC0_2e$: Rate per $C0_2$ emissions; $CfC0_2$: Correction factor of $C0_2$emissions.

Table 7 present ISV ($C0_2$ emissions) values for diesel vehicles (CISV – Decreto de Lei nº 82-D/2014, 2015).

Table 7 - ISV (CO_2 emissions) for diesel vehicles

$C0_2$ emissions category (g/Km)	Rate per $C0_2$ emissions (€.Km/g)	Correction factor (€)
Up to 95	19.97	1586.51
96 to 120	57.15	5173.80
121 to 140	126.75	13642.70
141 to 160	140.96	15684.40
More than 160	193.61	24137.71

For diesel vehicles with an emission of particles higher than 0.002 g/km the ISV has an increase of 500 €.

The ISV (C02 emissions) for diesel vehicles is also calculated using equation 2. The total ISV is presented by equation 3 (CISV - Decreto de Lei nº 82-D/2014, 2015):

$$TISV = ISV(CO_2e) + ISV(Es) \qquad (3)$$

TISV: Total ISV

2.2 Circulation Tax (IUC) and Value added tax (VAT)

IUC is an annual tax. For new vehicles the tax can be paid 90 days after the vehicle registration. It evaluates the same two elements: $C0_2$ emissions (environmental impact) and engine size. The following tables demonstrate the tax calculation for different values of $C0_2$ emissions and engine size. The following data is related to light passengers vehicles registered after 1 July 2007. Table 8 present IUC (Engine size) values (CIUC - Decreto de Lei nº 82-B/2014, 2015). There is an additional tax that is considered only for diesel vehicles.

Table 8 - IUC (Engine size)

Engine size category (cm3)	Tax`s value (€)	Additional tax for diesel vehicles (€)
Up to 1250	28.15	5.02
1250 to 1750	56.50	10.07
1750 to 2500	112.89	20.12
More than 2500	386.34	68.85

Table 9 presents IUC (C02 emissions) values.

Table 9 - IUC (CO_2 emissions)

$C0_2$ emissions category (g/Km)	Tax`s value (€)
Up to 120	57.76
120 to 180	86.55
181 to 250	187.96
More than 250	321.99

IUC has also a coefficient that is related to vehicle`s age (see Table 10).

Table 10 - IUC coefficient

Year of acquisition	Coefficient
2007	1.00
2008	1.05
2009	1.10
≥2010	1.15

The total IUC is represented by the next equation:

$$TIUC = Coef \times \left(IUC\left(Es \right) + IUC\left(C02\ e \right) \right) \quad (4)$$

TIUC: Total IUC; Coef: Coefficient

For diesel vehicles we need to add the additional tax (table 8) to equation 4 in order to determine the IUC. The Value added tax falls upon the ISV plus the vehicle price. The tax rate is 23% (Orçamento de Estado 2015 - Decreto de Lei nº 82-B/2014, 2015). This tax is deductible for income tax purposes. Equation 5 explains the procedure.

$$VAT = (VP + ISV) \times 0.23 \quad (5)$$

IVA: Value added tax; VP: Vehicle`s price

2.3 Taxes` reduction for rent-a-car companies

For rent-a-car companies the ISV has a discount of 40% if the vehicle has an hybrid engine (CISV - Decreto de Lei nº 82-D/2014, 2015). For other type of vehicles, there is also a tax reduction of 40%, if the vehicle respects the following criteria (CISV - Decreto de Lei nº 82-D/2014, 2015).

1. The vehicles should have a CO_2 emission level until 120g/km, confirmed by the certificate of conformity.
2. The undertaking rental companies should be licensed for the exclusive purpose of renting cars. The vehicles with tax reduction should not be rented or given for a time superior to 3 months to the same person or entity, for a period of 12 months in a row, nor could they be object, in the period of burden, of renting or assignment to people or juridical entities.
3. The renting should be named by contract, and the vehicles should travel together with the documents emitted by the rental company, that must identify the renter, his address and the renting time period.

2.4 Replacement constraint for rent-a-car companies

In legal terms, Portuguese rent-a-car companies are obliged to have vehicles with no more than 5 years after the first vehicle's registration (Decreto-Lei nº 207/2015, 2015).

3 Replacement Approaches

This topic will provide a review about models used in assets` replacement problems. First, it will contain models based on financial criteria and then on financial and non-financial criteria.

3.1 Discounted Cash Flows Methods

This sector will explain the different discounted cash flow methods that are used to evaluate assets `replacement

3.1.1 Equivalent Annual Cost (EAC)

This method is used in deterministic environments. EAC methodology uses normally operating costs, maintenance costs, investment values and salvage values to calculate annual costs. To achieve Equivalent Annual Cost the method applies an interest rate to the Cash inflow/outflow in order to discount back to its present value (PV). Then the method transforms the Cash-Flows in an annuity value. The equipment that should be replaced is called the defender and the new equipment that should replace the old is called the challenger. The method compares EAC of the defender equipment with EAC of the challenger equipment for the nth periods (n). When the old equipment and the new equipment are the same, the optimal replacement timing is when EAC is minimum, but when the equipment are different (defender and challenger) the solution is more complex. To define a critical replacement timing the method needs to balance EAC of the challenger equipment and the EAC to maintain the defender equipment. The EAC tends to decrease with an higher economic lifetime of the equipment. (Riggs, J. et al., 1997)

3.1.2 Net Present Value (NPV)

NPV is also a method used to evaluate assets `replacement. It uses positive and negative Cash-Flows and an interest rate representing the capital cost. The objective is to discount all the cash-flows back to its present value and compare NPV of both assets. For identical replacement the chosen period (economic lifetime) corresponds to the one that maximizes the NPV (with revenues as positives cash flows and costs as negative cash flows).

To understand if the replacement should be considered the analysis needs to determine the NPV for both equipment (for not identical replacement). If the challenger equipment has a higher NPV than the defender equipment the replacement must occur. If the lifetime of the assets is different, the analysis needs to replicate the lifetime of the asset with lower lifetime In order to compare assets using the same time horizon. But to analyze the replacement process considering that the replaced equipment is different from the new equipment it is important to analyze in which period the company will gain more value with the replacement. The higher NPV related to a replacement period determines optimal replacement level. In some cases (particularly for not identical replacement and for options with different maturities) an Annualized Net Present Value is defined in order to perform an analysis similar to the EAC; the only difference is that considers the revenues related to the vehicles and not only the costs (Filho & Kopittke, 2007; Guthrie, 2009).

3.1.3 Cost-Benefit Analysis

The Cost-Benefit analysis performed by Hynes (2005) to study aircraft`s design replacement included several environmental criteria. The main goal was to compare a baseline-aircraft with other possible options. The model created used several design variables (such as horizontal tail area and initial cruise altitude), several constraints (such as cruise range and stability margin), two parameters (number of engines and number of passengers) and three objectives (estimated ticket, fuel burn price, and Take-off weight). The purpose was to minimize the objectives using an optimized mathematical approach and to calculate the changes in operating costs and damage cost (environmental costs). After that, Real Option Theory was applied to study the problem using uncertainty related to historical data. The damage costs were calculated using social costs (externalities), this means that it was necessary to evaluate the pollution gases impact to the society.

A Cost-Benefit analysis can be very useful if the objective is to study a replacement; an aircraft`s design replacement, an equipment`s replacement or even a vehicles` replacement. But it is more difficult to study the replacement time and level using this approach. So, this method can be helpful to reinforce the replacement study, this means that can be important to verify if the replacement should or not occur.

3.1.4 Average Annual Cost (AAC)

Other possible methodology used in deterministic replacement problems is AAC. The annual cost is an average between all the costs at the nth period of the equipment. Wade & Boman (2003) presented an AAC methodology to analyze an engine replacement. They used a capital recovery factor and the engine present value to calculate amortization values. To determine average opportunity costs the study presented an annual average investment (that includes salvage value, depreciation cost and engine present value) multiplied by a primary interest rate. The annual total cost is defined as the sum of the maintenance annual cost, taxes annual cost, amortization value and the average opportunity cost. If the AAC of the new engine is lower than the AAC of the actual engine the replacement must occur (Wade & Boman, 2003; Valverde & Resende, 1997).

3.2 Dynamic Programming (DP)

DP is used in deterministic and uncertainty environments and it`s more complex than EAC, NPV and AAC. Dynamic Programming is applied in complex problems that have constraints and/or that use more inputs than the presented before (operating costs, maintenance costs, investment values and residual values) such as annual revenues (Dreyfus & Law, 1977), (Beveridge & Schecht, 1970) (Marques et al., 2005). In these cases, the optimal function is about maximizing the profits and not minimizing the costs (EAC and AAC).

The first investigations of stochastic replacement models were performed using a dynamic programming formulation (Bellman, 1955). DP for uncertainty environments assumes that for each parameter there is an associated probability, so each parameter as an associated uncertainty. This method is very useful because it can use different parameters and variables, so it`s more flexible and can generate more appropriated solutions (Marques et al., 2005; Hastings, 1968; McArthur, 1975; Chinneck, 2010).

3.3 Multi-criteria

The methodology MAQ (Feldens, et al, 2010) was created to analyze a buses` fleet replacement using economic criteria and non-economic criteria. MAQ is a mixed between other methodologies such as Multi Attribute Utility Theory (MAUT), Analytical Hierarchy Process (AHP) and Quality Function Deployment (QFD). Multi Attribute Utility Theory uses the expert`s opinion to score different criteria. Then the scores are summed using normalized weights. The best alternative is the one with the best score. Analytical Hierarchy Process is a tool that is used to complement Multi-Criteria decisions. The objective is to compare each criterion and to quantify the alternative`s importance. If the results comparison between Option A and B is 1, this means that both options have the same importance. If it`s 7, this means that Option A is 7 times more important than Option B.

Quality Function Deployment is a method used to perform design quality, in other words, to deploy quality functions and to set up methods for achieving the design quality. The objective of QFD is to quantify and qualify the perception of quality. To score non-economic results MAQ uses a utility function that was achieved by the interaction between MAUT, AHP and QFD methodologies. To analyze economic criteria it was used EAC method and replacement timing. The final results were presented as a Cost-Benefit analysis, the costs were evaluated using minimum EAC and the Benefits` score was evaluated using MAQ. Benefits and Costs had a different importance. The final score for each option (fleet of buses) was a weighted percentage between Benefits and Costs.

This approach can also be useful to study replacement problems and it calculates the replacement timing. This value was only influenced by economic criteria; this means that the replacement timing was not related to non-economic criteria. To find the value for replacement timing the method of replacement uses the Equivalent Annual Cost (EAC).

3.4 Real Options Theory (ROT)

In deterministic environments, there is no uncertainty associated to the salvage value, to operating costs and maintenance costs. Economic and competitive interactions originate cash flows structures different from what was initially estimated. Therefore, discounted-cash-flow (DCF) approaches to vehicles` replacement have a gap because they cannot properly evaluate real economic conditions (Zambujal-Oliveira & Duque, 2010; Amram & Kulatilaka, 1999). For that reason, Stochastic Dynamic Programming and Real Options Theory appeared. The objective of recent papers that studied these theories is to create practical and usable models in assets` replacement management (Adkins & Paxson, 2008; Trigeorgis, 1996).

The ROT`s methods thatp are most used are the closed-form solutions, partial-differential equations, and the binomial lattice (Mun, 2002). The closed-form solutions are models of equations that can be solved given a set of input assumptions. They are precise, fast, and easy to implement with the knowledge of basic programming but can be difficult to explain because the stochastic calculus involved is complex. Closed-form solutions also tend to be specific so they have a low flexibility. Black-Scholes model - Black & Scholes (1973) - was one of the first closed-form solutions created to study the price of options. Partial-differential equations models are used to solve problems when the variables are associated to mathematical functions. ROT uses variables that are associated to probability distribution functions (uncertainty), because of that models such as partial-differential equations can be considered.

Binomial lattices are based on binomial distribution and uses event trees to search for the best solution. They are easy to implement and to explain. Binomial lattices can be used in different problems because they are highly flexible. But two possible weaknesses are the significant computing power and time-steps to obtain good approximations that are needed. There are two ways to apply Binomial lattices, risk-neutral approach and replicating portfolio approach. The results obtained by the use of binomial lattices tend to approach those derived from closed-form solutions (Mun, 2002; Broyles, 2002).

To use ROT an historical cost analysis must be performed in order to identify patterns and to assess input parameters. To generate paths for stochastic variables one of the most used methods is Monte Carlo - simulation technique based on repeated random sampling to compute their results. This outputs provided by MC simulations will permit to estimate the future cash-flows and will also be used to fill the Real Options Analyses (ROA) (Pridgen, 1968).

Recently, some papers studied new approaches for Real Options Theory that reinforced the theory strength and its position as one of the methods most used in Replacement problems (Hynes, 2005; Zambujal-Oliveira & Duque, 2010).

3.5 Integrated equipment's replacement

Literature study replacement decisions for different types of equipment. Ji & Kite-Powel (1999) study replacement processes for ships` fleets, Valverde & Resende (1997) and Borgert et al. (2006)

for automobiles, Feldens et al. (2010) for buses` fleets and Hynes (2005) for aircraft`s design. There are also different equipment`s replacement strategies, which are chosen in order to respect and to follow the company`s strategy. (Filho & Kopittke, 2007) created some types of replacement: discharge without replacement, identical replacement, not identical replacement, replacement concerning technological progress and strategic replacement.

Discharge without replacement is used when equipment is no longer economically viable or useful to the company. This means that the equipment is discharged without being replaced. Identical replacement refers to replacement in which the replaced equipment is the same as the new equipment.

Not identical replacement is used when the replaced equipment is different from the new equipment. Replacement concerning technological progress refers to replacement that considers technological evolution; this means that new equipment normally are more efficient than older equipment. In order to respect this principle, the definition of obsolescence cost was created. Strategic replacement concerns to the impact in the production that an equipment`s replacement could create. Replacement of equipment that manufactures products with less quality must be penalized in the analysis. In order to do that, the analysis created a cost that reflects the negative impact on the revenues; this is explained because a better machine can manufacture better products with greater revenues.

It`s important to enhance that replacement concerning technological progress and strategic replacement reflect non-economic criteria (such as technological progress and equipment quality) in the replacement analysis using "artificial costs". It is also possible to find other approaches for the replacement decision. Valverde & Resende (1997) and Borgert et al (2006) used a model based only on economic criteria but other authors such as Feldens et al (2010) and (Hynes, 2005) used a Multi-criteria model (economic and non-economic criteria). Some multi-criteria models used traditional analysis for economic criteria, like EAC (equivalent annual cost) and the experts` opinion to evaluate non-economic criteria. These criteria are combined in a Cost-Benefit analysis (Feldens et al., 2010).

Studies applied for equipment`s design used optimization models with different parameters and constraints to define the optimized values for design variables (Hynes, 2005). Other studies used a comparison between multiple objectives of interest to the decision maker that are evaluated using normalized scores (Ismail Erol, 2003). In spite of the diverse studies about multi-criteria problems, the models are related only to the choice of one option to replace another one. So, the replacement time of decision is not evaluated or if it`s evaluated it only depends on economic criteria. Methodologies such as Multi Attribute Utility Theory (MAUT), Analytical Hierarchy Process (AHP), Function Deployment (QFD) and MAQ have been used to solve multi-criteria problems (Feldens et al., 2010; Collan & Liu, 2003; Ismail Erol, 2003; Roy & Vincke, 1983).

The traditional analysis of equipment`s replacement commonly calculates deterministic future cash flows to obtain indicators such as NPV (net present value) and EAC (equivalent annual cost). The economic lifetime approach considers the EAC for a range of possible lifetimes (Filho & Kopittke, 2007). EAC has to different components: the capital recovery cost and the equivalent annualized total cost (Sepulveda,et al.,1984). However, these analyses do not consider the uncertainty. Given this limitation a new tool for investment analysis appeared, the Theory of Real Options.

According Avinash & Pindyck (1994) three aspects characterize an investment`s projects: irreversibility, uncertainty and flexibility. The irreversibility is about being partial or completely impossible to recover the initial investment. The uncertainty is also important because the vehicle has a cost structure that is not completely predictable, so it has some uncertainty. The flexibility is also one of the conditions because the investor can sell the equipment without time restrictions; this means that he can always sell the equipment at the replacement timing. The real options theory appeared initially related to futures market, however it can also be used for valuating long term investment. Recently, some studies developed new approaches and applications that emphasize the importance of real options theory to solve replacement problems using Partial differential equation. Graeme Guthrie (2009) established a discounted cash flow analysis and a real option analysis to study a machinery replacement problem using Binomial Lattices trees (Guthrie, 2009; Minardi, 2004; Zambujal-Oliveira & Duque, 2010).

Different approaches can also be applied to deterministic and uncertainty environments, such as Dynamic Programming, which is considered a flexible and suitable method for more complex problems. Dynamic Programming uses more data than EAC but as a similar procedure for replacement evaluation (Marques et al., 2005). Regarding impacts assessment, some studies have demonstrate that examining environmental issues provides better information needed to take more conscientious decisions, this means that in order to perform a replacement decision it`s important also to analyze environmental criteria (Hynes, 2005).

3.5.1 Timing and Level of Replacement

Equipment's replacement is an important issue that can influence companies` financial and economic results. Therefore, it`s important to consider replacement timing in order to improve equipment`s economic efficiency. An equipment`s early replacement can result in a short life cycle with a considerable impact on recovery of the invested capital and a late replacement can drastically reduce the equipment`s salvage value (Valverde & Resende, 1997).

Replacement`s studies show how to calculate the optimal life of equipment, which is usually defined as the time horizon between the equipment`s service start and the time when it should be disabled and replaced for economic reasons. Commonly, the operational costs of equipment increase as its condition deteriorates over time; this means that older equipment have higher operating costs. When the cost reaches a certain level, the investment in new equipment becomes a better economic solution than maintaining the old equipment, designated by optimal replacement level. The time at optimal level is called optimal replacement timing. As a result of that, a basic replacement analysis usually study the tendency between operating cost and the net cost of replacement (capital recovery cost), which can be defined as the difference between the price of the new equipment and the resell value of the old (Ji & Kite-Powel, 1999).

3.6 Vehicles' Fleets Replacement and CO_2 Environmental Evaluation

One of the first papers written about vehicles` fleets replacement was "A Study in Equipment Replacement" - Eilon, King, & Hutchinson (1966) that explained how to achieve optimal replacement of fork lift trucks. This work used two methodologies, the first is about minimum average costs per truck per year, and the second was related to the approach of discounted cash flow.

Avramovich et al. (1982) created a linear programming model to manage a trucks` fleet. The model evaluated what type of tractors should be sold or traded in each week. The system was implemented in order to predict what could happen if some conditions occurred and to analyze the related consequences such as the financial impact. Other important work about fleets` replacement used a dynamic programming algorithm to optimize the projected discounted cash flow for individual highway tractors (Waddell, 1983). In the following years, this method was also applied to passengers` cars and light trucks.

In the 90`s several works appeared. Karabakal et al. (1994) developed a branch-and-bound algorithm based on Lagrangian relaxation methodology. In the following years Kabir (1996) and Scarf & Bouamra (1999) used a dynamic programming approach and a mathematical model based on the age of replacement, respectively. The goal`s models were to achieve minimization of the total discounted cost per unit time. Integer programming formulation for a parallel replacement with multiple challengers available was a new type of problem that was solved by (Keles & Hartman, 2004). These works used only the fleet's costs as single criteria to solve the replacement`s problems. For vehicles` fleets replacement most of the methodologies created was based on complex mathematical approaches.

In recent years, new and stronger approaches to equivalent annual cost literature have appeared. The importance of EAC methodology has emerged as one of the methodologies most used in equipment` replacement and vehicles` replacement. The study performed by Duarte et al. (2007) used EAC to analyze a buses` fleet replacement, Borgert et al. (2006) used EAC to analyze vehicles` fleet replacement and Oliveira (2000) used EAC to study a trucks` fleet replacement. In these studies the replacement method used is only about identical replacement. Also recently, a multi-criteria methodology (MAQ) was created to solve a buses` fleet replacement problem. MAQ used also EAC method to calculate economic criteria. (Feldens, Muller, Filomena, Neto, Castro, & Anzanello, 2010). Several authors studied and evaluated the environmental impact of $C0_2$ emissions. Most of them deal with emission costs as public costs. This public cost consists on the financial assessment of the damage caused by $C0_2$ emissions ("artificial cost") (Barbir, 2007). Social cost of $C0_2$ is difficult to analyze. The literature shows a disagreement about what value to consider for $C0_2$ social cost. Tol (2008) presents a literature review of 211 estimates of the social cost. The author, according literature review, found that the mean estimates of the social cost of CO_2 vary between \$24 and \$35 per metric ton of CO_2 (\$/tCO_2, in 1995 dollars).

Another important definition that is used to evaluate technological replacement is the CO_2 abatement cost. This cost try to evaluate the net costs to society per unit of CO_2 avoided, and usually it uses the investment (to buy the new vehicle), the net present value (lifetime fuel costs saving) and the lifetime CO_2 reduction. It was considered a constant average annual mileage of 16,000 km and an average vehicle lifetime of 13 years (Smokers, et al., 2006). In today`s world there is a new environmental paradigm. $C0_2$ emissions have an extreme importance and are related to a variety of protocols and deals, such as Kyoto Protocol (EESC, 2010). Due to that, higher vehicle`s $C0_2$ emissions led to new measures in order to oppose that tendency. In European Union was created in 2005 a new passenger car tax that results on a tax increasing for vehicles that have higher CO_2 emissions. The increasing taxes related to environmental impacts try to penalize consumers for the “bad environmental choice” and also try to incorporate social costs into vehicles` price (CEC, 2005).

4 Methodology

This chapter will describe and explain the deterministic and stochastic models used in this analysis. The first topic will provide an explanation about the deterministic model and the second topic will provide an explanation about the stochastic model.

4.1 Replacement costs specification

This chapter will describe and explain the deterministic and stochastic models used in the replacement analysis. The first topic will provide an explanation about discounted cash flows models and the second one will approach the stochastic model.

4.1.1 Maintenance costs

The maintenance costs will consider the estimated scheduled maintenance, this means that will be considered the costs to maintain performance of factory-recommended items at periodic mileage and/or calendar intervals. Maintenance costs are related to all the items that need to be replaced in order to maintain the vehicle`s performance and safety. For instance, the replacement of the oil filter, air filter, clutch fluid and brake fluid.

The estimated expense of scheduled maintenance costs of the case study vehicles are defined in Table 11. The expenses correspond to a 1 year of operation or 15,000 km traveled (Toyota, 2012). To define increasing maintenance costs this study will use aggregate maintenance costs considering the following values. There is also the assumption that each vehicle of the case study travels 15,000 km per year.

Table 11 - Annual maintenance costs for each vehicle

Vehicle / Year	1	2	3	4	5
Toyota Auris 1.6	128.38 €	194.38 €	162.66 €	194.38 €	128.38 €
Toyota Auris 1.8	129.12 €	197.84 €	171.26 €	197.84 €	129.12 €
Toyota Auris 2.0	160.07 €	268.05 €	190.60 €	292.87 €	160.07 €

4.1.2 Inspection and insurance costs

Inspection costs for rent-a-car vehicles follow the same procedure of the private vehicles without prejudice to the general system applied to the vehicles automobile inspections. The renting vehicles without driver must have inspections for verifying their commodity and safety conditions in the following situations (Decreto-Lei nº 144/2012, 2015) :

a) When they are allocated to the activity, unless they are registered vehicles in the name of the company with less than 180 days related to the vehicle's registration;

b) When they have suffered an accident that obligates to a long interruption of the vehicle's usage.

The mechanical review schedule is defined by: the first scheduled inspection is 4 years after the vehicle registration. After the first inspection the vehicle must be inspected once every 2 years. After the 8th year of registration the inspection must be annually performed. The inspection cost is 30.54€ (2015 values)

For estimating insurance costs, this analysis considered a simulation with eleven types of scenarios. The criteria used to establish the insurance costs are presented in tables 12 and It was considered the date of birth, date of driving license, number of years without accidents and number of accidents in the last 5 years – see table 12. (Império Bonança, 2012).

Table 12 - Criteria values used on the simulation

	Date of birth	Date of driving license	Number of years without accidents	Number of accidents in the last 5 years
Type 1	01/01/1987	01/01/2007	5	-
Type 2	01/01/1987	02/01/2007	2	1
Type 3	02/01/1972	02/01/1992	15	-
Type 4	03/01/1972	03/01/1992	7	-
Type 5	04/01/1972	04/01/1992	5	-
Type 6	05/01/1972	05/01/1992	3	1
Type 7	06/01/1972	06/01/1992	3	2
Type 8	01/01/1957	01/01/1977	30	-
Type 9	02/01/1957	02/01/1977	15	-
Type 10	03/01/1957	03/01/1977	4	1
Type 11	04/01/1957	04/01/1977	4	2

The costs related to each type of scenario are presented in table 13 and are only related to insurance against third party liability. According general rental terms the customer have the option to buy an insurance related to the renting period and is responsible for eventual damages resulting from the vehicle utilization (ARAC, 2015). Thereby this study assumes the assumption that eventual damages resulting from the vehicles utilization are totally supported by customers.

Table 13 - Annual insurance costs related to each type of scenario and each case study vehicle

	Auris 1.6	Auris 1.8	Auris 2.0
Type 1	271.68 €	259.57 €	317.46 €
Type 2	293.53 €	280.48 €	343.02 €
Type 3	200.61€	191.82 €	234.02 €
Type 4	200.61€	191.82 €	234.02 €
Type 5	200.61€	191.82 €	234.02 €
Type 6	243.72 €	233.02 €	284.35 €
Type 7	263.32 €	251.75 €	307.22 €
Type 8	200.61 €	191.82 €	234.02 €
Type 9	200.61 €	191.82 €	234.02 €
Type 10	235.88 €	225.54 €	275.22 €
Type 11	263.32 €	251.75 €	307.22 €

There is also the assumption that the risk associated with vehicle`s insurance is stable during time and that the insurance is paid at the end of the period.

4.1.3 Depreciation costs

Vehicles have a limited useful life and are considered depreciable assets, originating a depreciation cost in each accounting period. Higher depreciation costs tend to decrease companies` taxes. For that reason countries have depreciation procedures that present legal and permitted depreciations for each period. This analysis will consider two possible tax procedures (RAD, 2009) (Blank, 2014). The first procedure is called "Straight-line method" and it uses constant depreciation percentages for each period. The percentage defined for vehicles is 25% and it is applied to the vehicles` price plus ISV. The IUC is not considered because it can be paid until 90 days after the vehicle registration and it is an annual tax (CIUC - Lei nº 82-B/2014, 2015). The following equation shows the procedure:

$$DC = VP \times 0.25 \qquad (6)$$

DC: Depreciation cost per year; VP: Vehicles` price (including ISV)

The second procedure is called declining balance method and it uses higher depreciation percentages. This percentage is defined by a coefficient related to assets` expected life. The coefficient is 1.5 if the expected life is less than 5 years. The coefficient is 2 if the expected life is 5 or 6 years. The coefficient is 2.5 if the expected life is greater than 6 years. The depreciation rate for each coefficient can be calculated using equation 7 and the depreciation cost is calculated considering equation 8.

$$D\% = Coef \times 0.25 \tag{7}$$

D%: Depreciation rate; Coef: Coefficient of the declining balance method

$$DC = (VP - AD) \times D\% \tag{8}$$

DC: Depreciation cost per year; D%: Depreciation rate; VP: Vehicle`s price; AD: Accumulated depreciation per year

For a coefficient equal to 1.5 the depreciation percentage is 37.5%; with a coefficient equal to 2 the depreciation percentage is 50%; for a coefficient equal to 2.5 the depreciation percentage is 62.5%. This procedure follows also equation (6) to calculate depreciation cost. But if the ratio between book value and remaining expected life is greater than the chosen depreciation ratio (0.375 or 0.5 or 0.625) the company can use as depreciation rate the linear straight rate. For diesel and gasoline vehicles there is a depreciation cost limit of 25.000 €. For vehicles that only use electric energy there is depreciation cost limit of 62.500 €. Considering plug-in hybrid vehicles there is depreciation cost limit of 50.000 € and for vehicles that use a GPL or GNV system there is a depreciation cost limit of 37.500€. (Decreto de lei n.º 4/2015, 2015)

4.1.4 Investment Value and Salvage value

The investment values (vehicle`s price) used in the analysis comes from a list of acquisition prices provided by Toyota (Toyota, 2012). To estimate the salvage value, the analysis used estimated market values that were established by vehicles` experts which constituted references to the market buyers and sellers. These values are determined using vehicle`s dealerships information about the vehicles that were sold. They consist on an average price considering that information (Guia do Automóvel, 2012; Toyota, 2012). The following tables present the estimated market values for each case study vehicle. These values don`t include taxes (VAT and ISV) – see Table 14 for the investment value (new vehicles).

Table 14 – Investment value for each case study vehicle
Toyota Auris 1.6: *Toyota Auris 1.6 WT-i Exclusive (similar to Toyota Auris 1.6 Exclusive); Toyota Auris 1.8HSD: *1 Toyota Auris 1.8HSD; Toyota Auris 2.0: *2Toyota Auris 2.0 D-4D Exclusive

Type of vehicle	investment value (€)
Toyota Auris 1.6	17068*
Toyota Auris 1.8	19165*1
Toyota Auris 2.0	18313*2

Table 15 presents the salvage value for each vehicle.

Table 15 - Salvage values for each case study vehicle
Toyota Auris 1.6: *Toyota Auris 1.6 WT-i Exclusive (similar to Toyota Auris 1.6 Exclusive); Toyota Auris 1.8HSD: *[1]Toyota Auris 1.8 HSD; Toyota Auris 2.0: *[2]Toyota Auris 2.0 D-4D Exclusive; *[3] Toyota Auris D-4D Sol

Type of vehicle	1 year of utilization (€)	2 years of utilization (€)
Toyota Auris 1.6	20340*	18380*
Toyota Auris 1.8	20330*[1]	18450*[1]
Toyota Auris 2.0	25530*[2]	21250*[3]

4.1.5 Cost of Capital

To establish discount rate this analysis uses the cost of capital and the inflation rate. Weighted Average Cost of Capital (WACC) can be defined as the following equation (Brealey & Myers, 2006):

$$WACC_n = kd \times (1-t) \times \frac{FK}{V} + ke \times \frac{Veq}{V} \quad (9)$$

$WACC_n$: Nominal Weighted Average Capital Cost; kd: Cost of debt; t: Firm`s average tax rate; FK: Market value of debt; V: Market value of total capital; ke: Cost of equity; Veq: Market value of equity

To determine cost of equity for companies listed on the stock exchange it is possible to use CAPM (Capital asset pricing model) that is defined in equation 10 (Brealey & Myers, 2006):

$$ke = Rf + \beta \times (Rm - Rf) \quad (10)$$

ke: cost of equity; Rf: risk-free rate of return; Rm: market return; β: beta of stock

Where beta is defined by the following equation:

$$B = \frac{Cov\ (r_i : r_m)}{Var\ (r_m)} \quad (11)$$

Cov (r_i : r_m):Covariance between the return of a company i and the return of a market portfolio m; Var (r_m): Variance of the market return

For companies not listed on the stock exchange it`s possible to find an equity beta using betas of comparable companies that are publicly traded. In order to do that, it is important to remove the effect of capital structure on the betas. Higher amount of debt produces higher variability in earnings (financial leverage) and an higher sensitivity to the stock prices. To unlever the betas it`s possible to use equation 12 (Damodaran, 2005):

$$B_{unlevered} = \frac{B_{levered}}{(1+(1-t)) \times \frac{FK}{Veq}} \quad (12)$$

t: Firm`s average tax rate; FK: Market value of debt; Veq: Market value of equity

The equation 12 is a conventional approach and assumes that debt carries no market risk (has a beta of zero), this means that the debt level is not going to change the investment`s volatility over time. (Damodaran, 2005). Using the unlevered Betas it`s necessary to determine a beta averaged value. After that, it`s important to adapt the achieved beta to the capital structure of the company (case study company). In order to do that, the equation 12, can be used again to find a levered beta that will be used as the equity beta. To resume the previous procedure, this study presents the following guideline (Damodaran, 2005):

1. Find the business that your company operates in.
2. Find companies listed on the stock exchange that have the same businesses and obtain their betas.
3. In order to remove the effect of capital structure, find the unlevered beta for each company found on section 2.
4. Compute an average between the unlevered betas.
5. In order to adapt (to lever) the beta to the capital structure of the company that is being analyzed, find the levered beta. This beta can be defined as the equity beta.

Following a similar procedure it is possible to determine cost of debt for (equation 13) (Brealey & Myers, 2006):

$$kd = Rf + Rp \quad (13)$$

kd: Cost of equity; Rf: Risk-free rate of return; Rp: Risk premium

Sometimes for companies that are not listed on the stock exchange it is possible to consider the ratio between the net result and the equity to evaluate *ke* and the ratio between the financial cost and the debt to evaluate *kd*. However, these ratios use book values to determine cost of equity and cost of debt. Considering a Discount Cash Flows analysis, these values should not be considered as the first option. The firm`s average tax rate considered in this analysis is 21% (Orçamento de Estado 2015 - Decreto de Lei nº 82-B/2014, 2015). Other important rate that needs to be considered is the inflation rate. In 2015 the predicted inflation rate is 0.7% (Orçamento de Estado 2015 - Decreto de Lei nº 82-B/2014, 2015). It is considered that this rate is constant during the evaluation period. The relation between $WACC_r$, $WACC_n$ and inflation rate is defined by equation 14. This study will use a real WACC.

$$(1+WACC_n) = (1+WACC_r) \times (1 + i_{inf}) \quad (14)$$

$WACC_n$: Nominal WACC (Discount rate); $WACC_r$: Real WACC; i_{inf}: Inflation rate

4.2 Free cash flow to the firm discount model

This topic presents the deterministic model used in this study. First it will explain the Equivalent Annual Cost model and after that the Cash Flow analysis. The last subtopic will present the calculation of the deterministic model using EAC and the Cash Flow analysis.

4.2.1 Free cash flow to the firm

For this model the propose objective is to determine the replacement from the firm`s point of view (rent-a-car), this means that it needs to consider Free Cash Flows to the Firm (FCFF). For total costs this analysis will consider vehicles` maintenance costs (MC), insurance costs (INS) and inspection costs (INSP). Vehicle`s fuel costs were not considered because there is the assumption that they are supported by rent-a-car customers. Depreciation costs (DC), investment value (vehicles` price) - (I) and taxes related to vehicles (ISV and IUC) were also considered. VAT was not considered because it is deductible for income tax purposes. The residual value (RV) is defined as equation 15:

$$RV = SV - (SV - BV) \times t \qquad (15)$$

RV: Residual value; SV: Salvage value; BV: Book Value; t: Firm`s average tax rate

Therefore, the free cash flows to the firm equation is shown in the following equation (Brealey & Myers, 2006):

$$\begin{aligned} FCFF(x) = & -\left(I(0) + ISV(0)\right) \;\; + SV(n) - (SV(n) - BV(n)) \times t \;\; + \\ & \left(-MC(x) - INS(x) - INSP(x) - DC(x) - IUC(x)\right)(1-t) + DC(x) \end{aligned} \qquad (16)$$

x: period; n: last period; I(0): Investment value; SV: Salvage value; BV: Book Value; MC: Maintenance cost; INS:
x: period; n: last period; I(0): Investment value; SV: Salvage value; BV: Book Value; MC: Maintenance cost; INS:
Insurance cost; INSP: Inspection cost; DC: Depreciation costs; ISV(0), IUC: Taxes; t: Firm`s average tax rate

The previous equation can be simplified using equation 17

$$\begin{aligned} FCFF(x) = & -\left(I(0) + ISV(0)\right) + SV(n) \times (1-t) \\ & + (BV(n) + DC(x)) \times t - \left(MC(x) + INS(x) + INSP(x) - IUC(x)\right)(1-t) \end{aligned} \qquad (17)$$

x: period; n: last period; I(0): Investment value; SV: Salvage value; BV: Book Value; MC: Maintenance cost; INS: Insurance cost; INSP: Inspection cost; DC: Depreciation costs; ISV(0), IUC: Taxes; t: Firm`s average tax rate;
Insurance cost; INSP: Inspection cost; DC: Depreciation costs; ISV(0), IUC: Taxes; t: Firm`s average tax rate

4.2.2 Equivalent annual cost model

The chosen model to analyze deterministic environment is Equivalent annual cost. This option was chosen because it is the most used for replacement problems and for cost analysis problems. The revenues obtained from vehicle rental will not be considered because there is the assumption that they not change with the vehicle`s replacement. EAC is calculated by discounting back to its Present Value (PV) all the cash-flows from each year. EAC transforms PV in an annuity value and is defined in the next equation (Borgert, Hunttemann, & Schultz, 2006) (Brealey & Myers, 2006).

$$EAC(n)=\left(I(0)+\sum_{x=1}^{x=n}\frac{OCF\ (x)}{(1+i)^{x}}(A/P,i,n)\right)-\left(RV(n)(A/F,i,n)\right) \tag{18}$$

EAC: Equivalent annual cost; A/P: Annuity factor; A/F: Sinking fund factor; i: Discount rate (Capital Cost); x: Period; n: Last period; I(0): Investment value (vehicles` price); OCF: Operational cash-flows; RV: Residual value

Using the preceding equation (equation 18) and also equation 15, this study will define EAC using the Cash Flows defined in equation 17. This is shown in the following equation:

$$EAC(n)=((I(0)+ISV(0)+\sum_{x=1}^{x=n}\frac{\left(MC(x)+INS(x)+INSP(x))+IUC(x)\right)(1-t)-t\times DC(x)}{(1+i)^{x}})(A/P,i,n)-((SV(n)(1-t)+BV(n)\times t)(A/F,i,n)) \tag{19}$$

EAC: Equivalent annual cost; A/P: Capital recovery factor; A/F: Sinking fund factor; i: Discount rate; x: Period (year); n: Last period; I(0): Investment value (vehicles` price); SV: Salvage value; BV: Book Value; MC: Maintenance cost; INS: Insurance cost; INSP: Inspection cost; DC: Depreciation costs; ISV, IUC: Taxes; %IVA: Percentage of value added tax; t: Firm`s average tax rate

In order to establish positives EAC`s the preceding equation (equation 19) signs (plus and minus) were changed. This modification was done to enable a more accurate balance between different EACs.

Where A/P is defined by equation 20:

$$(A/P,i,n)=\frac{i}{1-(1+i)^{-n}} \tag{20}$$

A/P: Annuity factor; i: Discount rate (Capital Cost); n: last period

And A/F is defined by equation 21:

$$(A/F,i,n)=\frac{i}{(1+i)^{n}-1} \tag{21}$$

A/F: Sinking fund factor; i: Discount rate (Capital Cost); n: last period

Other important definition that needs to be considered is the Capital recovery cost (CR). CR balances the Investment Cost with the Residual Value. This is shown in equation 22 (Sepulveda, Souder, & Gottfried, 1984), (Filho & Kopittke, 2007):

$$CRC(n)=I\ (A/P,i,n)-RV\ (A/F,i,n) \tag{22}$$

CR: Capital recovery cost; I: Investment value (vehicles` price); RV: Residual value; A/P: Annuity factor; A/F: Sinking fund factor; n: last period

Another important definition is EATC (Equivalent annualized total cost). EATC can be defined as the following equation (Sepulveda, Souder, & Gottfried, 1984), (Filho & Kopittke, 2007):

$$EATC(n)=\sum_{x=1}^{x=n}\frac{OCF\ (x)}{(1+i)^{x}}(A/P,i,n) \tag{23}$$

EATC: Equivalent annualized total cost; OCF: Operational cash-flows (include Maintenance cost, Insurance cost, Inspection cost, Depreciation costs and Circulation Tax); A/P: Annuity factor; n: last period; x: period

Using CR and EATC it is possible to define EAC using equation 24 (Sepulveda et al, 1984; Filho & Kopittke, 2007):

$$EAC(n)=CRC(n)+EATC(n) \tag{24}$$

EAC: Equivalent annual cost; CR: Capital recovery cost; EATC: Equivalent annualized total cost; n: last period

For this study, the propose objective is to determine the replacement of different vehicles There is the assumption that the replacement can occur after one year of utilization of the defender vehicle and that in the fifth year of utilization the company is obliged to replace the vehicle (legal constrain). The replacement procedure works in the following way (Brealey & Myers, 2006; Filho & Kopittke, 2007):

- The estimation of the challenger EAC considers a given maturity of five years. Thus, the vehicle will have an economic lifetime of five years, not considering a replacement before the fifth year (for the challenger vehicle).
- Primarily, the defender EAC is calculated considering the economic lifetime of the defender vehicle. In the first period, it considers one year of past operation and four more years of future operation. This means that in each period the defender EAC will be determined considering the free cash-flows to the firm of future operation (cash-flows to maintain the defender vehicle). For the defender vehicles the investment value considered in each period

(vehicles` age) is the residual value of the vehicle. This happens because the residual value is the economic value that the rent-a-car company would receive if the vehicle was sold in that period.

The replacement procedure compares the $EAC_{defender}$ with the $EAC_{challenger}$. It considers that EATC (Equivalent annualized total cost) of the defender increase during time. The replacement procedure compares the EAC in each period (each period is related to the vehicle's age of the defender) - when the $EAC_{defender}$ > $EAC_{challenger}$ the defender vehicle should be immediately replaced by the challenger vehicle and when $EAC_{defender}$ < $EAC_{challenger}$ the defender vehicle shouldn't be replaced in that period. The first period when the $EAC_{defender}$ is higher than the $EAC_{challenger}$ is the critical timing of replacement. Equation 25 represents the $EAC_{defender}$ calculation:

$$EAC(xa) = ((SV(xa) \times (1-t) + BV(xa) \times t))(A\,/\,P, i, n-xa) + \sum_{x=xa+1}^{n} \frac{(MC(x) + INS(x) + INSP(x) + IUC(x)) \times (1-t) - t \times DC(x)}{(1+i)^{x-xa}} (A\,/\,P\,, i, n-xa) - ((SV(n) \times (1-t) + BV(n) \times t))(A\,/\,F, i, n-xa) \quad (25)$$

A/P: Capital recovery factor; A/F: Sinking fund factor; i: Discount rate; n: Last period (n=5); xa: vehicle`s age of the defender (1≤xa≤4); SV: Salvage value; BV: Book Value; MC: Maintenance cost; INS: Insurance cost; INSP: Inspection cost; DC: Depreciation costs; ISV, IUC: Taxes; t: Firm`s average tax rate;

Equation 26 represents the $EAC_{challenger}$ calculation:

$$EAC = ((I(0) + ISV(0))(A\,/\,P, i, n) + \sum_{x=1}^{n} \frac{(MC(x) + INS(x) + INSP(x) + IUC(x)) \times (1-t) - t \times DC(x)}{(1+i)^{x}} (A\,/\,P, i, n) - \big((SV(n) \times (1-t) + BV(n) \times t)\big) \times (A\,/\,F, i, n) \quad (26)$$

A/P: Capital recovery factor; A/F: Sinking fund factor; i: Discount rate; n: Last period (n=5); I(0): Investment value (vehicles` price); SV: Salvage value; BV: Book Value; MC: Maintenance cost; INS: Insurance cost; INSP: Inspection cost; DC: Depreciation costs; ISV(0), IUC: Taxes; t: Firm`s average tax rate; xa: vehicle`s age of the defender (1≤xa≤5)

The $EAC_{challenger}$ included in equation 26 has always the same value because the maturity of the challenger vehicle is always equal to 5 years. The critical replacement level is the EAC related to the critical replacement timing. If the decision is to replace the defender vehicle the critical replacement level is the $EAC_{challenger}$ but if the decision is to don´t replace the defender vehicle the critical replacement level is the $EAC_{defender}$ in the first period (EAC to maintain the defender vehicle until the last year).

4.3 Real Options model

This section will explain the stochastic model used in this analysis. This analysis will explain two ways to apply Real Options and binomial trees: risk neutral approach and replicating portfolio approach (Copeland & Antikarov, 2001), (Guthrie, 2009).

Binomial trees are based on an “up state” and a “down state”, this means that the project uncertainty produces in each node an expected Value of the underlying risk asset for each state and a payoff with flexibility called Call option. The “up movement” and the “down movement” are determined using Monte Carlo outputs. The following sections will explain risk-neutral probabilities and replicating portfolio. This topic will also explain Monte Carlo technique in order to find project volatility that is needed to measure uncertainty and to produce Binomial trees. The last section will provide an explanation about real options applied to the case study (Copeland & Antikarov, 2001), (Guthrie, 2009).

4.3.1 Monte Carlo simulation method

To apply Monte Carlo this section will present three steps to apply the method (Copeland & Antikarov, 2001).

1. **Monte Carlo inputs.**

Step 1 defines the variable with uncertainty and the Monte Carlo input. First, this step defines a probability distribution in each period related to the variable with uncertainty. Lognormal and Normal are candidates because they are the most used in this type of analysis. After that, step 1 needs to establish the mean and the standard deviation of the variables with uncertainty in each period. These values are the Monte Carlo inputs. A suitable approach for volatility is to use the quotient between the standard deviation and the average value (absolute values).

2. **Define forecast variable.**

This step defines the forecast variable whose distribution will be simulated by the Monte Carlo method. The objective of step 2 is to define the mean and the standard deviation for the cost of capital considering the uncertainty defined in Step 1. In order to achieve that the following equations will be used (27, 28 and 29).

$$z = \ln\left(\frac{PV_1 + FCFF_1}{PV_0}\right) \tag{27}$$

$$PV_0 = \sum_{t=1}^{n}\left(\frac{FCFF_t}{(1+WACC)^t}\right) \tag{28}$$

$$PV_1 = \sum_{t=2}^{n} \left(\frac{FCFF_t}{(1+WACC)^{t-1}} \right) \tag{29}$$

z: capital cost with uncertainty; PV_0: Present value in period zero (considering five more years of future operation); PV_1: Present value in the first period (considering one year of past operation and four more years of future operation; $FCFF_1$: Free Cash Flows in period one WACC: Capital cost

The variable *z* defines the percentage changes in the value of the project from one time period to the next where the present value at period 1 and period 0 (present value of the project) can be defined using equations 28 and 29. $FCCF_1$ represents the Free Cash Flow at period 1. The present value at 0 is constant in equation 27 (deterministic value). These present values don`t include the investment value.

The variable *z* represents the cost of capital considering uncertainty. To define the mean and the standard deviation of *z*, Monte Carlo method performs a number of trials (this analysis consider 10,000 iterations). These trials use the distributions defined in step 1 to determine 10,000 values for the uncertainty variable in each period and then to produce 10,000 values for the PV_1. After that, using equation 27 is possible to determine 10,000 values for the variable z. The values will be used to calculate the mean and the standard deviation for the variable *z*. The standard deviation of variable *z* is considered as a good estimator for the project volatility.

3. Building the event tree.

With volatility this analysis can establish the “up and down movements” using the next equations - 30 and 31 (Copeland & Antikarov, 2001).

$$u = e^{(\delta\sqrt{T})} \tag{30}$$

$$d = 1/u \tag{31}$$

u: Up movement; d: Down movement; ʆ: Volatility; T: length of time between nodes

With u and d a binomial tree can be build using the “up movement” and “down movement” and the probabilities of the tree. The following sections will show two approaches to determine the probabilities of the tree and to understand how to use it.

4.3.2 Replicating portfolio approach

RPA (Replicating portfolio approach) considers a portfolio composed of “twin securities” whose values have the same payoffs of the project. Using “Law of one price” it is possible to say that “to prevent arbitrage profits, two assets that have exactly the same payout in every state of nature are perfect substitutes, therefore, have exactly the same prices or value” (Copeland & Antikarov, 2001).

Considering a portfolio of *m* shares of the “twin security” and *B* bonds the method replicates the project`s payoffs (C_u and C_d) in “up state” and “down state” (equation 32 and 33). The expression $B \times (1 + r_f)$ defines the project payout, considering a risk-free payout.

$$RP_{up\ state} : m \times V_u + B \times \left(1 + r_f\right) = C_u \tag{32}$$

$$RP_{down\ state} : m \times V_d + B \times \left(1 + r_f\right) = C_d \tag{33}$$

RP: Replicating portfolio payoff; m: Shares; B: Bonds; V_u: Value of the underlying risk asset (“up state”); V_d: Value of the underlying risk asset (“down state”); r_f: Risk free rate; C_u: Call option (“up state”); C_d: Call option (“down state”)

Calculating equation 32 minus equation 33 it is possible to determine the *m* value that it is also called hedge ratio (equation 34).

$$m = \frac{\left(C_u - C_d\right)}{\left(V_u - V_d\right)} \tag{34}$$

m: Hedge ratio; V_u: Value of the underlying risk asset (“up state”); V_d: Value of the underlying risk asset; C_u: Call option (“up state”); C_d: Call option (“down state”)

And knowing *m*, *B* can also be calculated using equation 32 or equation 33. With *B* and *m* it is possible to determine the call option value (C_o) using the following equation:

$$m \times V_0 + B_0 = C_0 \Leftrightarrow m \times V_0 - C_0 = B_0 \tag{35}$$

m: Shares; B_0: Risk free payout; V_o: Value of the underlying risk asset; C_0: Call option;

It is difficult to find a twin security whose cash payoffs in every state of nature are perfectly correlated with the projects payoffs. Because of that, MAD (Market asset disclaimer), (Copeland & Antikarov, 2001) assumes that it is possible to use the project present value instead of the “twin security”. So, values V_u and V_d can be determined using the traditional Cash-Flow analysis and the “up movement values” / “down movement values”. Table 16 demonstrates the calculation for each node (s) considering 3 periods (t):

Table 16 - Value of the underlying risk asset for each node and period (considering three periods)

V (s,t)	0	1	2	3
0	V_0	$V_0 \times u$	$V_0 \times u^2$	$V_0 \times u^3$
1		$V_0 \times d$	$V_0 \times d \times u$	$V_0 \times d \times u^2$
2			$V_0 \times d^2$	$V_0 \times d^2 \times u$
3				$V_0 \times d^3$

The node V_0 is the present value of the project. Each node has a related probability called *q* for upper nodes and *(1 - q)* for lower nodes. For instance, Figure 2 shows a binomial tree with the first six nodes and with probabilities *q* and *1-q*. These probabilities are the real probabilities related to the "up movement" and "down movement" and are calculated using Monte Carlo outputs. The values *q* and *(1-q)* are the same for all the tree nodes.

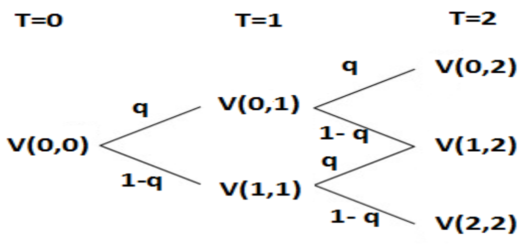

Figure 1 - Binomial tree

To determine Present Value of the project (PV), RPA also uses *WACC*, *q* and *(1-q)* and the Value of the underlying risk asset. Equation 36 shows the calculation of the expected present value for Figure 2 tree.

$$PV = \frac{V(0,2) \times q^2 + 2 \times V(1,2) \times (1-q) \times q + V(2,2) \times (1-q)^2}{(1+WACC)^2} = V(0,0) \tag{36}$$

V: Value of the underlying risk asset; q: Probability "Up movement"; (1-q): Probability "Down movement"; WACC: Weighted average capital cost

Equation 36 shows that there are three assumptions that must be mutually consistent with each other: the present value, the objective probabilities multiplied by the payoffs and the risk-adjusted discount rate.

4.3.3 Risk neutral approach

There is a second Real Options approach that is very simple and easy-to-use. It`s called Risk neutral approach. RNA considers a hedge portfolio composed of one share of the underlying risk asset (V_0) and a short position in *m* shares of the option that is being priced (call option) (Copeland & Antikarov, 2001). RNA establishes a hedge ratio *m* in order to produce a risk free portfolio over the next short interval of time. If V_0 goes down the lost value is offset by the gain on the short position in the call option. The hedge ratio is calculated by equating the payoffs in "up state" and "down state". If the project`s payoffs are the same in both "states" this means that the portfolio is riskless (equation 37 and 38). The following equation considers also MAD assumption (Copeland & Antikarov, 2001).

$$u \times V_0 - m \times C_u = d \times V_0 - m \times C_d \tag{37}$$

$$m = \frac{(u-d) \times V_0}{C_u - C_d} \tag{38}$$

m: Shares; B: Bonds; V_0: Value of the underlying risk asset; r_f: risk free rate; C_u: Call option ("up state"); C_d: Call option ("down state"); d: "down movement", u: "upper movement" ; $u = e^{(\sqrt{\sigma T})}$; $d = 1/u$

Following the same procedure it is possible to define the portfolio Present Value (equation 39).

$$PPV = V_0 - m \times C_0 \tag{39}$$

PPV: Portfolio present value; V_0: Value of the underlying risk asset; m: Shares; C_0: Call option

And considering the risk-free rate that produces a risk-free payout it is possible to define that (equation 40):

$$\left(V_0 - m \times C_0\right) \times \left(1 + r_f\right) = u \times V_0 - m \times C_u$$

Or (40)

$$\left(V_0 - m \times C_0\right) \times \left(1 + r_f\right) = d \times V_0 - m \times C_d$$

m: Shares; V_0: Value of the underlying risk asset; r_f: risk free rate; C_u: Call option ("up state"); C_d: Call option ("down state"); C_0: Call option; V_0: Value of the underlying risk asset; d: "down movement", u: "upper movement"

Considering hedge portfolio equations (38 and 40) it is also possible to define the call option equation (equation 41).

$$C_0 = \left(C_u \times \left(\frac{(1 + r_f) - d}{u - d} \right) + C_d \times \left(\frac{u - (1 + r_f)}{u - d} \right) \right) \div (1 + r_f) \tag{41}$$

C_0: Call option; r_f: Risk free rate; C_u: Call option ("up state"); C_d: Call option; d: "Down movement", u: "Up movement"; $u = e^{(\sqrt{\sigma T})}$; $d = 1/u$

The result of the Call Option obtained using RPA is similar to the result obtained using RNA. RPA and RNA are similar approaches that in theory produce similar results. One of the differences is that RPA uses real probabilities but RNA uses "risk free" probabilities that adjust payoffs to risk. The probability *p* and (1-p) can be defined using equation 42. The calculation is shown in the next equation:

$$p = \left(\frac{(1 + r_f) - d}{u - d} \right)$$

$$\left(1 - p\right) = \left(\frac{u - (1 + r_f)}{u - d} \right) \tag{42}$$

p: Probability "upper movement" (risk-free); (1-p): Probability; d: "Down movement" (risk-free); u: "Up movement" (risk-free)"; r_f: Risk free rate

The probabilities can be used to fill the Binomial Lattice tree explained in Figure 2. The procedure is the same. Using RNA, probability q is replaced by probability p. Another important difference between RPA and RNA is the discount rate. RPA uses *WACC* and RNA uses a discount free rate (r_f). In spite of that the determined PV_0 is equal using RNA or RPA (Copeland & Antikarov, 2001) To calculate the same expected present value of equation (34) using RNA it is possible to use equation 43:

$$PV = \frac{V(0,2)\times p^2 + 2\times V(1,2)\times(1-p)\times p + V(2,2)\times(1-p)^2}{(1+r_f)^2} = V(0,0) \qquad (43)$$

V: Value of the underlying risk asset; p: Probability "Up movement" (risk-free); (1-p): Probability "Down movement" (risk-free); r_f: Risk free rate

RNA is commonly used to evaluate payoffs where we don`t know the risk-discounted rate and the objective probabilities.

4.3.4 Model application to determine replacement timing and level

This topic explains the Real Options model used in this analysis to evaluate vehicles` replacement. The real options model was created using as reference (Copeland & Antikarov, 2001).

The goal of this model is to find a feasible approach to evaluate uncertainty considering historical data related to the utilization of a vehicle (defender vehicle). The data related to the challenger vehicle is considered deterministic because it is considered that the challenger vehicle has never been used before.

This study will consider uncertainty related to the variable maintenance costs. This means that the variable maintenance costs will have a probability distribution that will be used as input of the Monte Carlo simulation.

The FCFF related to the defender vehicle will be used to define a binomial tree. The Risk neutral approach will be used in this analysis.

The following equations are used to compute the Monte Carlo simulation and find the expected volatility of the project – standard deviation of the variable *z* considering uncertainty related to the maintenance costs - see section 4.3.1. These equations are applied to the defender vehicles.

$$PV_0 = \sum_{x=1}^{x=n} \frac{((MC(x)+INS(x)+INSP(x)+IUC(x))(1-t)-t\times DC(x)}{(1+i)^x} - \frac{((SV(n)(1-t)+BV(n)\times t)}{(1+i)^n} \qquad (44)$$

$$PV_1 = \sum_{x=2}^{x=n} \frac{((MC(x)+INS(x)+INSP(x)+IUC(x))(1-t)-t\times DC(x)}{(1+i)^{x-1}} - \frac{((SV(n)(1-t)+BV(n)\times t)}{(1+i)^{n-1}} \qquad (45)$$

$$\text{FCFF}(1) = ((MC(1) + INS(1) + INSP(1) + IUC(1))(1-t) - t \times DC(1) \quad (46)$$

PV_0: Present value in period zero (considering five more years of future operation); PV_1: Present value in the first period (considering one year of past operation and four more years of future operation; $FCFF_1$: Free Cash Flows in period one; i: Discount rate; x: Period (year); n: Last period; I: Investment value; SV: Salvage value; BV: Book Value; MC: Maintenance cost; INS: Insurance cost; INSP: Inspection cost; DC: Depreciation costs; ISV, IUC: Taxes; t: Firm`s average tax rate

The project volatility is used to find the "up probabilities" and "down probabilities" for the "up state" and "down state" of the recombined tree of the aggregated FCFF.

The following procedure will be used to evaluate the replacement process considering Real Options.

1. Build the tree for the uncertainty variable.

The first step is to build a binomial tree related to the values of the uncertainty variable. To do that we will use the volatility related to the uncertainty variable to produce the "up movement" and "down movement" (see equations 30 and 31 – section 4.3.1). Using the uncertainty variable value in the first period and the "up movement" and "down movement" we will determine in each period the "up state" and "down state" of the tree. – see equation 47 and 48.

$$VV_{(s,t+1)} = V_{(s,t)} \times u \quad (47)$$

$$VV_{(s+1,t+1)} = V_{(s,t)} \times d \quad (48)$$

s: Node – s ≥ 0; t: Period – t ≥ 1; u: "up movement" where $u = e^{(\delta\sqrt{T})}$; d:: "down movement" where $d = 1/u$ - VV: Variable value

2. Build the tree for the FCFF.

Using the deterministic values (taxes, insurance costs, inspection costs, depreciation costs and salvage value) and the values defined before (uncertainty variable values for the "up state" and "down state" of the maintenance costs– see topic 1) we can fill each node with the FCFF related to the defender vehicle. It`s important to understand that the investment value is not yet considered and that the salvage value is only considered in the last period.

3. **Recombine the tree starting from the last node to determine the value of the future cash-flows**

Using the FCFF tree (see topic 2) we will start from the last node to recombine the tree using the "up probabilities -p" and "down probabilities – (1-p)". These probabilities will be determined using the project volatility (cost of capital volatility) and the RNA approach. Equation 49 shows the calculation for each node of the tree. The last nodes maintain the same values – FCFF in the last period.

$$V_{(s,t)} = \frac{(V_{(s,t+1)} + FCFF_{(s,t+1)}) \times p + (V_{(s+1,t+1)} + FCFF_{(s+1,t+1)}) \times (1-p)}{(1+r_f)} \qquad (49)$$

s: Node – s ≥ 0; t: Period – t ≥ 1; r_f: Risk-free rate; FCFF: Free cash-flow to the firm; p: RNA probability; V: Value of the future cash-flows

4. **Define the EAC in each node**

The values of the future free cash flows to the firm that were defined above (see topic 3) will be converted into an EAC. This is necessary because we want to compare vehicles with different maturities. In order to do that equation 50 will be used. The investment value related to defender vehicle in each period (residual value) will also be considered.

$$EAC_{(s,t)} = (RV_t + V_{(s,t)}) \times A/P(r_{f,} \, n - t) \qquad (50)$$

s: Node – s ≥ 0; t: Period – t ≥ 1; r_f: Risk-free rate; V: Value of the future cash-flows; EAC: Equivalent annual cost considering future cash-flows; A/P: Annuity factor; n: last period (n=5); RV: Residual Value (used as investment value)

The $EAC_{(s,t)}$ defined by equation 50 returns the Equivalent Annual Cost to maintain the defender vehicle until the last period of the tree (n) considering uncertainty related to project.

5. **Decision of replacement**

The procedure that will be used to evaluate the decision of replacement is to compare the $EAC_{challenger}$ considering the deterministic analysis and the EAC of equation 50 ($EAC_{defender}$ considering uncertainty). Equation 51 shows the procedure to determine if the defender vehicle should be replaced or not in each node.

$$If\{\, EAC_{defender(s,t)} \; - \; EAC_{challenger} \leq 0\}$$ – don`t replace the vehicle

(51)

$$If\{\, EAC_{defender(s,t)} \; - \; EAC_{challenger} > 0\}$$ – replace the vehicle

s: Node – s ≥ 0; t: Period – t ≥ 0;

With this procedure we can compare the $EAC_{challenger}$ with the $EAC_{defender}$ with uncertainty.

5 Case Study Analysis

This chapter analyzes a numeric case considering the variables related to the replacement process: maintenance costs, inspection costs, insurance costs, investment value, salvage value, depreciation costs, and taxes. Afterwards, it will estimate critical timing and replacement level using the case study data. It will be used deterministic and stochastic models that were defined earlier (sections 4.2.2 for the deterministic model and 4.3.4 for the stochastic model). In order to validate the evaluation, this analysis will also introduce a sensitivity analysis to determine the importance of each variable, including the costs related to $C0_2$ emissions.

5.1 Replacement information for different costs

This topic estimates the value of the variables that will be used to determine critical timing and replacement level. It will also be estimated the cost of capital that is going to be used in this study considering a real WACC.

5.1.1 Maintenance costs

In order to determine the maintenance costs, this analysis assumed the estimated maintenance costs defined on chapter 4 (sector 4.1.1) and created a model to predict future values on a deterministic environment using a linear regression. To define increasing maintenance costs this study used aggregate costs in order to assume that during time the maintenance costs tend to increase. The equation 52 formalizes the maintenance costs for the vehicle – Auris 1.6 (defender vehicle) and defines the aggregate costs for that vehicle.

$$MC_{Auris\ 1.6} = 163.46x \ with \ \mathrm{x} \geq 0 \tag{52}$$

MC: Maintenance cost; x: Period (year)

The following equation is for the vehicle – Auris 1.8 HSD (challenger vehicle) and defines the aggregate costs for that vehicle.

$$MC_{Auris\ 1.8} = 167.05x \ with \ \mathrm{x} \geq 0 \tag{53}$$

MC: Maintenance cost; x: Period (year)

Equation 54 is for the vehicle – Auris 2.0 (defender vehicle) and defines the aggregate costs for that vehicle.

$$MC_{Auris\ 2.0} = 215.95x \ with \ \mathrm{x} \geq 0 \tag{54}$$

MC: Maintenance cost; x: Period (year)

5.1.2 Inspection and insurance costs

The vehicle inspection costs were defined in chapter 4 (section 4.1.2). The first scheduled inspection is 4 years after the first vehicle registration. After the first inspection the vehicle must be inspected once every 2 years. After the 8th year of registration the inspection must be annually performed. The inspection cost is 30.54€ according the Portuguese law - (Decreto-Lei nº 144/2012, 2015). Regarding the simulation defined on chapter 4 (sector 4.1.2) about insurance costs, the case study considered an average value between the values of the different types of scenarios. Table 17 shows the insurance costs that are going to be considered on this study.

Table 17 - Insurance costs for each vehicle

	Defender vehicles		**Challenger vehicle**
	Auris 1.6	**Auris 2.0**	**Auris 1.8**
Insurance costs (€/year)	234.05	273.14	223.75

5.1.3 Taxation effects

This section determines the tax values related to vehicles tax (ISV), circulation tax (UC) and value added tax (VAT) supported by the legal framework referred in chapter 2. The following table shows the value of each component of ISV (CO_2e - CO_2 emissions and Es - Engine size) and the value of total ISV (the sum of each component) charged to each vehicle. Table 18 also shows the percentage of each ISV component considering the total ISV. ISV of Toyota Auris 1.8 HSD (challenger vehicle) has a reduction of 40% because this type of model is an hybrid vehicle (see section 2.3).

Table 18 - ISV components and total ISV - *40% reduction for hybrid vehicles; %=(ISV component/Total ISV)

	Defender vehicle				**Challenger vehicle**	
Tax\Vehicles	**Auris 1.6**	**%**	**Auris 2.0**	**%**	**Auris 1.8**	**%**
ISV (Es)	2 147.93 €	58%	4 027.93 €	51%	1852.76 €*	100%
ISV (CO_2e)	1 570.80 €	42%	3 848.80 €	49%	0 €	0%
Total ISV	3 718.73 €	100%	7 876.73 €	100%	1852.76 €*	100%

Table 19 shows the two components of the IUC (CO_2e - CO_2 emissions and Es - Engine size) and the total IUC charged to each vehicle. There is also a coefficient of 1.15 that is multiplied by the sum of each component and an additional tax for diesel vehicles (in this case – Toyota Auris 2.0) – see section 2.2.

Table 19 - IUC components and total IUC for each vehicle (coefficient=1.15 multiplied by the sum of each component);%=(IUC component/Total IUC); *gasoline vehicles

	Defender vehicles				Challenger vehicle	
Tax\Vehicles	**Auris 1.6**	**%**	**Auris 2.0**	**%**	**Auris 1.8 HSD**	**%**
IUC (Es)	56.50 €	34%	112.89 €	45%	112.89 €	58%
IUC (CO_2e)	86.55 €	53%	86.55 €	35%	57.76 €	29%
IUC – coefficient	21.46 €	13%	50.04 €	12%	25.60 €	13%
Additional tax – diesel	0 €*	0%	20.12 €	8%	0 €*	0%
Total IUC	164.51 €	100%	249.48 €	100%	196.25 €	100%

Table 20 shows the VAT values charged for each vehicle. The indicated values take in consideration the values defined in section 4.1.4 plus the ISV values that were defined above. For rent-a-car companies, VAT is deductible for income tax purposes, not being considered in the determination of the FCFF.

Table 20 - VAT for each vehicle

	Defender vehicles		Challenger vehicle
Vehicles	**Auris 1.6**	**Auris 2.0**	**Auris 1.8**
VAT	4 781.00 €	6 024.00 €	4 834.00 €

5.1.4 Salvage value

To determine the salvage value of each vehicle in the replacement process, the case study will consider the values defined on chapter 4 (section 4.1.4), not forgetting the ISV and VAT defined in section 5.1.3. This section considers an exponential regression in order to predict a salvage value equation. The next equation is for the vehicle - Auris 1.6 (defender vehicle).

$$SV_{Auris\ 1.6} = 25030\, e^{-0.165x} \ with\ x \geq 0 \tag{55}$$

SV: Salvage value; x: Period (year)

Using an exponential regression, this study determined the equation of salvage value for Auris 1.8 HSD (challenger vehicle) – see equation 56.

$$SV_{Auris\ 1.8} = 25242\, e^{-0.169x} \ \text{with } x \geq 0 \tag{56}$$

SV: Salvage value; x: Period (year)

Using an exponential regression, this study determined the equation of salvage value for Auris 2.0 (defender vehicle). – see equation 57.

$$SV_{Auris\ 2.0} = 31951\, e^{-0.208x} \ with\ x \geq 0 \quad (57)$$

SV: Salvage value; x: Period (year)

5.1.5 Depreciation costs

For estimating the depreciation costs generated by the vehicle, this study considered "Straight-line method" with a depreciation rate of 25% (RAD, 2009), meaning that after 4 years of operation, the vehicle will be fully depreciated (booking value equal to 0). Table 21 represents the depreciation costs for each vehicle, considering the investment value defined in section 5.1.4 (salvage value in period 0). The depreciation costs do not consider VAT because this tax is deductible for income tax purpose.

Table 21 - Depreciation costs for each vehicle

Vehicle	DC per year
Toyota Auris 1.6 (defender)	5 087.40 €
Toyota Auris 1.8 (challenger)	5 130.49 €
Toyota Auris 2.0 (defender)	6 494.11 €

5.1.6 Cost of Equity

This section explains the calculation of the variables used to determine the cost of equity. The Portuguese rent-a-car market doesn`t have any company listed on the stock market. Since less than 5% of the companies that operate in the rent-a-car market are publicly traded, it`s relevant to define a procedure for estimating the cost of equity for companies that aren`t listed on the stock market. To do that this study will use betas of comparable companies that are publicly traded. The cost of equity is determined using the variables: risk-free interest, market return and beta (see section 4.1.5).

This risk-free interest rate is based on the return provided by the treasury government bonds. The bonds should be chosen considering its maturity and compliance with the investment period. According with the market, the yield of the Portuguese bonds with a maturity of 5 years was 1.6% in January 2015 (Investing.com).

The market return is based on the growth of the rent-a-car market. The rent-a-car market has been recovering since 2013, after the financial crisis. In 2013 and 2014 the market growth was 8.4% and 4.0%, respectively. For 2015 the expected market growth is 5% (DBK, 2014).

The Beta determines the risk from exposure to general market movements; this rate compares the volatility of an investment with the volatility of the market. This study used the data from three rent-a-car companies that operate on the Portuguese market and that are publicly traded – Avis, Hertz and Europcar. The data is showed in Annex 2. To lever the beta this analysis used averaged values of Equity and Debt of the Portuguese rent-a-car market – see Annex 3. Table 22 summarizes the Beta computation and the average levered beta that is going to be used in this study. The Betas were found using equation 11 and 12 – section 4.1.5.

Table 22 - Beta variables and Average levered beta using the Portuguese rent-a-car market data

Variables	Values
AVIS Beta	0.07
HERTZ Beta	0.26
EUROPCAR Beta	0.03
Average unlevered beta	0.12
tax rate	21%
Average Equity (€)	900808.78
Average Debt (€)	4346267.72
Average levered Beta	0.58

Considering the values defined for the risk-free rate, beta and market return, the cost of equity can be determined, using equation 10 – section 4.1.5 – and represented by equation 58:

$$Ke = 1.6\% + 0.58 \times (5.0\% - 1.6\%) = 3.6\% \quad (58)$$

Table 23 contains values according Annex 3, and computes the ratio net income / equity, used to understand if a determined value for the cost of equity is appropriated, concerning the amount of net income returned as a percentage of equity.

Table 23 - Ratio net income / equity

Year	Ratio
2013	5.9%
2012	-22.2%
2011	-18.3%
2010	-
2009	-1.0%
2008	-4.3%
2007	5.5%
2006	4.5%
2005	-0.3%
2004	-0.5%

From Table 23, the negative ratios (negative net income) verified between 2008 and 2012 concerned to the financial crisis that occurred in Portugal. Considering only the periods between 2004 and 2007 and the year of 2013 the ratio has an average of 3%. The indicated value is similar to the cost of equity that was previously determined (3.6%), supporting the use of cost of equity as a rate of 3.6%

5.1.7 Cost of Debt

The cost of debt used in this analysis will be the average loan rate applied to Portuguese companies by Portuguese banks. This rate was 4.6% in January 2016 (Portuguese National Bank, 2015). Table 24 was determined considering Annex 4, and computes the ratio interests / obtained loans. This relation can be used for data matching purposes in order to understand if a determined value for cost of debt is suitable. The ratio interests / obtained loans is the amount of interests returned as a percentage of obtained loans.

Table 24 - Ratio interests / obtained loans

Year	Ratio
2013	5.5%
2012	4.9%
2011	4.4%
2010	5.1%

Considering the values between 2010 and 2013 (historical values) the ratio has an average of 5.0%. This value is similar to the cost of debt that was determined above (4.6%), for that reason this study used as cost of debt a rate of 4.6%.

5.1.8 Weighted Average Capital Cost

Considering the variables that were defined in section 5.1.6 and 5.1.7 the following equation represents the calculation of nominal WACC (equation 9 – section 4.1.5). The debt and equity values are related to 2014. These values are an average of the book values related to the Portuguese rent-a-car market (see Annex 2).

$$WACC_n = 4.6\% \times (1 - 0.21) \times \frac{4346267.72}{5247077.50} + 3.6\% \times \frac{900808.78}{5247077.50} = 3.6\% \qquad (59)$$

Considering the $WACC_r$ equation (equation 14 – section 4.1.5) equation 60 represents the calculation of real WACC. This rate is going to be considered as the discounted capital cost of this study.

$$WACC_r = \frac{1+3.6\%}{1+0.7\%} - 1 = 2.9\% \qquad (60)$$

It`s important to understand that this study used book values to estimate the market values *FK/V (*value of debt / value of total capital)*, Veq/V (*value of equity / value of total capital) and that the beta evaluation was defined using companies not listed on the Portuguese stock market. Because of that, this study included a Monte Carlo simulation with three uncertainty variables – *FK/V, Veq/V and Beta* – in order to understand the impact of these three variables on $WACC_r$. The simulation generated 10,000 iterations of $WACC_r$ considering random values for the uncertainty variables (*FK/V, Veq/V* and

Beta ranging between 0 and 1). The result for the $WACC_r$ was a mean of 2.7%, with a standard deviation of 0.6%. Therefore, this study will consider a WACC ranging from 2.1% to 3.3% (considering an interval related to one standard deviation – 1 sigma). Table 25 has the values of a minimum and maximum $WACC_r$ considering different values for *FK/V, Veq/V* – the result was a minimum $WACC_r$ of 2.86% and a maximum $WACC_r$ of 2.91% (yellow cells). This means that *FK/V* and *Veq/V* have a lower impact on WACC.

Table 25 - Simulation of WACCr using different values for FK/V (value of debt / value of total capital) and Veq/V (value of equity / value of total capital).

FK/V	*Veq/V*	$WACC_n$	$WACC_r$
0.1	0.9	3.58%	2.86%
0.2	0.8	3.58%	2.86%
0.3	0.7	3.59%	2.87%
0.4	0.6	3.60%	2.88%
0.5	0.5	3.60%	2.88%
0.6	0.4	3.61%	2.89%
0.7	0.3	3.62%	2.90%
0.8	0.2	3.62%	2.90%
0.9	0.1	3.63%	2.91%

Considering the simulation results (Monte Carlo simulation and the simulation presented in table 25), we can say that a $WACC_r$ of 2.9% (equation 60) is a suitable rate for the rent-a-car market. In order to improve the accuracy of the evaluation, this study will also consider a $WACC_r$ ranging from a rate of 2.1% to 3.3% - Monte Carlo output, assuming a constant WACC over a period of 5 years.

5.1.9 Numeric case

The following tables present the numeric values estimated for each vehicle (see table 26, 27 and 28 - not discounted to its present value). The salvage value in period 0 (investment value – vehicle`s price) includes only the vehicles tax (ISV). Table 26 shows the variables values for Toyota Auris 1.6 (defender vehicle).

Table 26 - Variables values for Toyota Auris 1.6

Toyota Auris 1.6 – defender						
V. Values (€) / Period (years)	0	1	2	3	4	5
IUC		164.51	164.51	164.51	164.51	164.51
MC		163.46	326.92	490.38	653.84	817.30
INS		234.05	234.05	234.05	234.05	234.05
INSP		0.00	0.00	0.00	30.54	0.00
SV	20350.00	21223.00	17995.00	15258.00	12937.00	10969.00
DC		5087.40	5087.40	5087.40	5087.40	0.00
BV		15262.20	10174.80	5087.40	0.00	0.00

Table 27 shows the variables values for Toyota Auris 1.8 (challenger vehicle).

Table 27 - Variables values for Toyota Auris 1.8 HSD - *considering the 40% reduction of ISV

Toyota Auris 1.8 – challenger						
V. Values (€) / Period (years)	0	1	2	3	4	5
IUC		196.25	196.25	196.25	196.25	196.25
MC		167.05	334.10	501.15	668.20	835.25
INS		223.75	223.75	223.75	223.75	223.75
INSP		0.00	0.00	0.00	30.54	0.00
SV	20522.00	21317.00	18002.00	15203.00	12839.00	10843.00
DC		5130.49	5130.49	5130.49	5130.49	0.00
BV		15391.46	10260.98	5130.49	0.00	0.00

Table 28 shows the variables values for Toyota Auris 2.0 (defender vehicle).

Table 28 - Variables values for Toyota Auris 2.0

Toyota Auris 2.0 – defender						
V. Values (€) / Period (years)	0	1	2	3	4	5
IUC		249.48	249.48	249.48	249.48	249.48
MC		215.95	431.90	647.85	863.80	1079.75
INS		273.14	273.14	273.14	273.14	273.14
INSP		0.00	0.00	0.00	30.54.00	0.00
SV	25976.00	25951.00	21077.00	17119.00	13904.00	11293.00
DC		6494.11	6494.11	6494.11	6494.11	0.00
BV		19482.32	12988.21	6494.11	0.00	0.00

5.2 Analysis of the replacement process

This chapter is going to analyze the deterministic (EAC) model and stochastic (Real Options) model in order to study the replacement process.

5.2.1 Replacement timing and level considering $C0_2$ emissions legal framework - discounted free cash flow analysis

To analyze the replacement process, this study will compute the EAC for each vehicle and compare two components: Capital recovery cost and Equivalent annualized total cost. These components will be determined considering the FCFF methodology that was defined on section 4.2.1. To simplify the calculation this study will analyze a fleet with only one vehicle. Table 29 shows EAC of the defender and challenger vehicles.

Table 29 - Defender and challenger EAC; xa - vehicle`s age of the defender (years)

xa	Years to maturity - defender	$EAC_{defender}$ - Auris 1.6 (€)	$EAC_{defender}$ - Auris 2.0 (€)	Years to maturity - challenger	$EAC_{challenger}$ - Auris 1.8) (€)
1	4 years to maturity	3241.08	4437.04	5 years to maturity	2702.41
2	3 years to maturity	3078.26	4126.46		
3	2 years to maturity	2942.30	3872.23		
4	1 year to maturity	2811.45	3647.22		
5	0 years to maturity	-	-		

The $EAC_{challenger}$ is equal in each period because its maturity is always the same (5 years). In the first period the $EAC_{defender\ (xa=1)}$ (for both defender vehicles) > $EAC_{challenger\ (xa=1)}$, meaning that in the first period (vehicle`s age of the defender equal to one), the best decision is to replace the defender vehicle for the challenger vehicle. For that reason the critical replacement timing is one year and the critical replacement level is 2702.41€ - yellow cell. The graphical analysis of EAC can be seen on the following graphic (see figure 2).

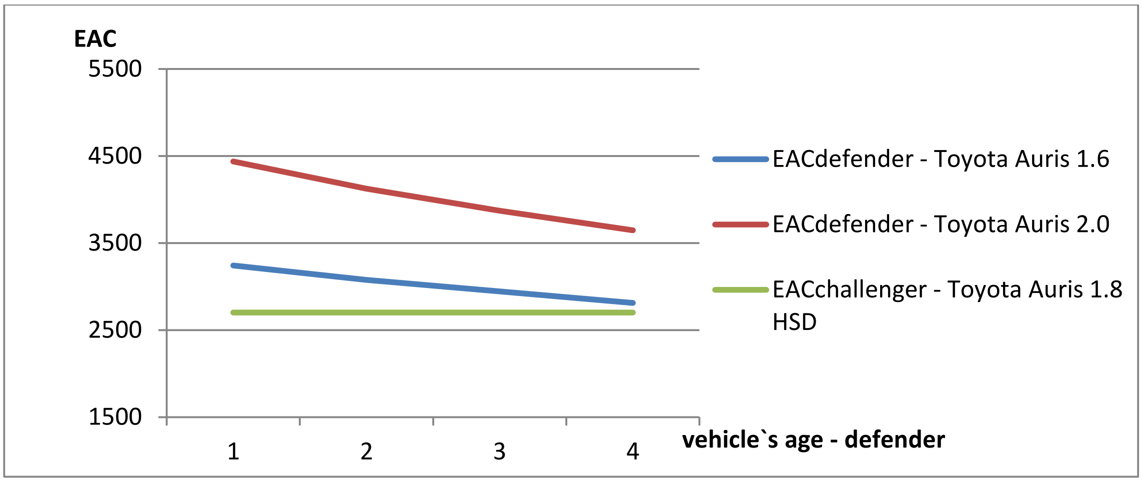

Figure 2 - EAC for defender and challenger vehicles

Comparing Toyota Auris 1.8 with Toyota Auris 2.0 the $EAC_{challenger}$ in each period is always lower than $EAC_{defender}$, this means that, even if the decision about replacement is delay, the best decision in each period is to replace the vehicle. Comparing Toyota Auris 1.8 with Toyota Auris 1.6 in each period, the $EAC_{challenger}$ is also always lower than $EAC_{defender}$. Figure 2 supports the $EAC_{defender}$ tendency to decrease along the time, with this tendency there are only to possible outcomes for the critical replacement timing: replace the defender vehicle in the first period or don` t replace it. Table 30 presents the Capital recovery cost (CRC) and Equivalent annualized total cost (EATC) values in each period and for each vehicle.

Table 30 - CRC (Capital Recovery Cost) and EATC (Equivalent Annualized Total Cost) for each vehicle; xa: vehicle`s age of the defender (years)

	CRC (€)			EATC (€)		
	Defenders		Challenger	Defenders		Challenger
xa	**Auris 1.6**	**Auris 2.0**	**Auris 1.8**	**Auris 1.6**	**Auris 2.0**	**Auris 1.8**
1	3285.52	4464.52	2851.62	-44.45	-27.48	-149.21
2	2963.65	3948.51		114.61	177.95	
3	2576.84	3372.27		365.46	499.96	
4	1850.92	2381.35		960.53	1265.87	
5	-	-		-	-	

For the challenger vehicle the CRC and the EATC are constant values along the time period (constant maturity). For the defender vehicles the CRC decreases along the time. This happens because the investment value also decreases along the time. The EATC for the defender vehicles increases along the time and it has a negative value in the first period because the effect of the aggregated depreciation costs in the cash-flows (average tax rate multiplied by the depreciation costs) is higher than the value of the remaining aggregated operational cash-flows. This means that the values of the depreciation costs tend to be higher than the other operational costs (maintenance, insurance, inspection and IUC). Considering the defender vehicles the CRC tendency to decrease is higher than the EATC tendency to increase, this explains the tendency of EAC to decrease its values along the time. CRC has greater values than EATC; this means that the EAC value is more sensitive to CRC variations than to EATC variations.

The vehicles (defender and challenger) have different maturities. For instance in the first period of the analysis the challenger vehicle has five years of maturity and the defender vehicle has four years of maturity. Table 31 shows the present value of the CRC (without the EAC factors – A/P and A/F) and the annualized CRC (CRC value used for the EAC calculation) considering a defender vehicle (Toyota Auris 1.6) with one year (four years to maturity).

Table 31 - CRC and Annualized CRC in period one for Toyota Auris 1.8 and Toyota Auris 1.6

First period - (vehicle`s age of the defender =1)		
Vehicle	**CRC Present Value (€)**	**Annualized CRC (€)**
Auris 1.8 – challenger	13096.97	2851.62
Auris 1.6 – defender	12241.88	3285.52

The previous table shows the difference of the CRC present value between the Toyota Auris 1.8 vehicle (challenger) and the Toyota Auris 1.6 vehicle (defender) and the difference of the annualized CRC between the Toyota Auris 1.8 vehicle (challenger) and the Toyota Auris 1.6 vehicle (defender). The CRC present value is higher for the Toyota Auris 1.8 vehicle (challenger) comparing with the Toyota Auris 1.6 vehicle (defender) but the annualized CRC is lower for the Toyota Auris 1.8 vehicle (challenger) comparing with the Toyota Auris 1.6 vehicle (defender). This difference is explained because the vehicles (defender and challenger) have different maturities. These different values show that EAC is lower with an higher maturity and the importance that the maturity has considering the EAC analysis that was defined.

5.2.2 Replacement timing and level without considering $C0_2$ emissions legal framework - discounted free cash flow analysis

To analyze the replacement process without considering CO_2 emissions this study will exclude the CO_2 emissions legal framework - ISV tax (CO_2 emissions) including the 40% discount of ISV (legal framework related to CO_2 emissions) and IUC tax (CO_2 emissions) – see section 2.1 and 2.2. The taxes IUC and ISV have two components: engine size and CO_2 emissions. The component related to CO_2 emissions will be excluded. This exclusion changes the variables: investment value, depreciation costs and IUC. Table 32 shows EAC results without considering legal framework related to CO_2 emissions.

Table 32 - Defender and challenger EAC in each period without considering legal framework related to $C0_2$ emissions; xa - vehicle`s age of the defender (years)

xa	Years to maturity - defender	$EAC_{defender}$ - Auris 1.6 (€)	$EAC_{defender}$ - Auris 2.0 (€)	Years to maturity - challenger	$EAC_{challenger}$ - Auris 1.8) (€)
1	4 years to maturity	3158.77	4349.41	5 years to maturity	2864.79
2	3 years to maturity	2997.19	4041.86		
3	2 years to maturity	2862.46	3790.63		
4	1 year to maturity	2732.82	3568.59		
5	0 years to maturity	-	-		

Comparing the contents of Table 32 and Table 29 (EAC table considering legal framework related to $C0_2$ emissions) the critical timing of replacement remains the same, the first period (xa=1 year). However, there is evidence of an increase of 6.0% of the critical replacement level - yellow cell. The ISV reduction of 40% is the most relevant parameter for this difference. The difference between $EAC_{challenger}$ and the $EAC_{defender}$ is lower. In the third period (xa=3 years) and last period (xa=4 years) the EAC of Toyota Auris 1.6 (defender vehicle) is lower than the EAC of Toyota Auris 1.8 (challenger vehicle). This means that considering Toyota Auris 1.6 as the defender vehicle the critical timing of replacement is the first period but if the decision about replacement is delay until the third period the replacement decision is to don't replace the defender vehicle.

Table 33 shows the variation in each period between the EAC without legal framework related to CO_2 emissions and the EAC with legal framework related to CO_2 emissions.

Table 33 - Variation between EAC without legal framework related to CO_2 emissions and EAC with legal framework related to CO_2 emissions - (EAC without $C0_2$ emissions – EAC with CO_2 emissions)/ EAC with CO_2 emissions; xa - vehicle`s age of the defender (years)

	EAC variation (%)		
	Defenders		Challenger
xa	Auris 1.6	Auris 2.0	Auris 1.8
1	-2.5	-2.0	6.0
2	-2.6	-2.1	
3	-2.7	-2.1	
4	-2.8	-2.2	
5	-	-	

The previous table shows that the $EAC_{challenger}$ without legal framework related to CO_2 emissions increases 6.0% and the $EAC_{defenders}$ decreases between 2.0% and 2.2% for Toyota Auris 2.0 and decreases between 2.5% and 2.8% for Toyota Auris 1.6. Because of that the difference between $EAC_{challenger}$ and the $EAC_{defender}$ is lower. The input relevant for this difference is the 40% reduction of the ISV (vehicles tax) for Toyota Auris 1.8 (hybrid vehicle – challenger). Table 35 shows the CRC and EATC without legal framework related to CO_2 emissions.

Table 34 - CRC (Capital Recovery Cost) and EATC (Equivalent Annualized Total Cost) for each vehicle without considering legal framework related to CO_2 emissions; xa: vehicle`s age of the defender (years)

	CRC (€)			EATC (€)		
	Defenders		Chalenger	Defenders		Challenger
xa	Auris 1.6	Auris 2.0	Auris 1.8	Auris 1.6	Auris 2.0	Auris 1.8
1	3219.13	4301.83	3118.73	-60.35	47.58	-253.94
2	2905.45	3805.91		91.74	235.95	
3	2533.80	3266.82		328.66	523.81	
4	1850.92	2381.35		881.90	1187.24	
5	-	-		-	-	

The values presented in the previous table have a similar behavior of the values presented in table 30 (EAC with legal framework related to CO_2 emissions). The CRC of the defender vehicles increases with legal framework related to CO_2 emissions but the CRC of the challenger vehicle decreases with legal framework related to CO_2 emissions. The 40% reduction of the vehicles tax (ISV) for the challenger vehicle explains this decreasing trend. For the defender vehicles the Book Value increasing explains the CRC increasing trend. This happens because for the defender vehicles the investment value is equal to the residual value. The EATC of Toyota Auris 1.6 and the EATC of the challenger vehicle Toyota Auris 1.8 are higher considering legal framework related to CO_2 emissions but the EATC of the defender vehicle Toyota Auris 2.0 is lower (except for xa=4). This difference can be explained analyzing the depreciation costs. For Toyota Auris 1.8 the depreciation cost decreases 5.7% and for that is expected an higher EATC. For Toyota Auris 1.6 the depreciation cost increases 8.4% but this higher deprecation is offset by the increase of the IUC. Considering Toyota Auris 2.0 the depreciation cost increases 17.4% and the IUC increasing is not enough to balance this value. This difference is not noted when xa=4 (Toyota Auris 2.0 with 4 years) because in this period the vehicle is completely depreciated (DC=0). This conclusion shows that considering legal framework related to CO_2 emissions the defender vehicle with an higher ISV tends to decrease its equivalent annualized total cost – EATC.

Figure 3 shows the EAC without CO_2 emissions legal framework for each vehicle.

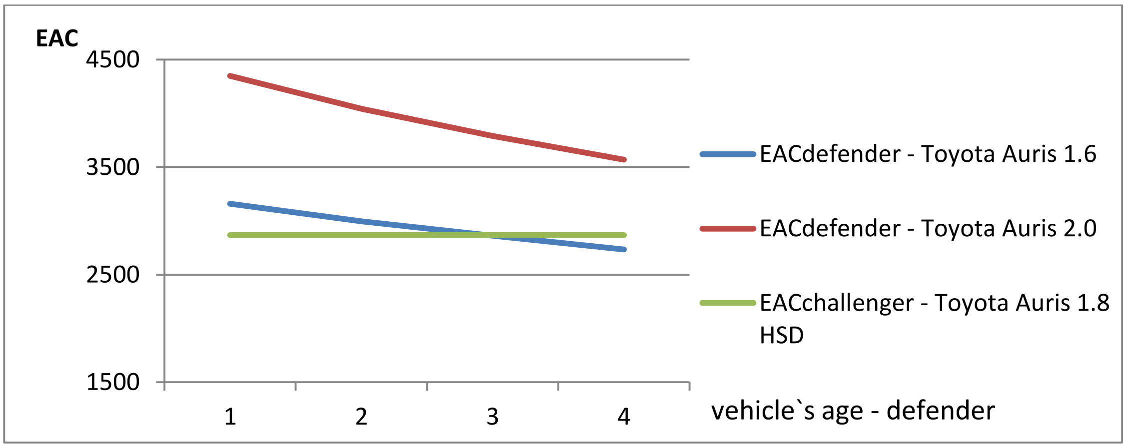

Figure 3 - EAC without CO_2 legal framework for defender and challenger vehicle

Figure 3 shows that the difference between the $EAC_{challenger}$ and the $EAC_{defender}$ is reduced without considering CO_2 legal framework and that the replacement process produces different results comparing with EAC with CO_2 emissions legal framework (see Figure 2).

5.2.3 Replacement process considering $C0_2$ emissions legal framework - real options approach

To analyze the process of replacement considering a stochastic environment, this study computed a Monte Carlo simulation to predict the values of the variables with uncertainty for both defender vehicles. The chosen uncertainty variable is the maintenance costs and the cost of capital rate is the risk-free (risk neutral approach).

The first goal of the Monte Carlo simulation is to estimate the expected volatility of the project (see sections 4.3.1 and 4.3.4). Initially for the maintenance costs it was considered a lognormal distribution, with a standard deviation of 30% (30% of the average value) in each period and an average equal to the deterministic value in each period (assumption used to create inputs to Monte Carlo simulation). The lognormal distribution was chosen because it is the distribution probability most used to evaluate costs. This happens because combinations of lognormal distribution are themselves lognormal and the values are always positive (Copeland & Antikarov, 2001). There is also the assumption that the distribution probabilities between the periods are not correlated.

There were generated 10,000 iterations for the maintenance costs in each period that originated 10,000 results for the variable *z* - *z* is considered as the cost of capital considering uncertainty (see section 4.3.1 to understand the variable z calculation). The variable *z* is determined using the stochastic values (maintenance costs) and the deterministic values (taxes, insurance costs, inspection costs, depreciation costs and salvage value). Table 35 resumes and exemplifies the Monte Carlo simulation that was defined for the Toyota Auris 1.6 - defender vehicle.

Table 35 - Monte Carlo simulation for Toyota Auris 1.6 (defender vehicle); MC: Maintenance Costs; $z = \frac{PV_1 + FCFF_1}{PV_0}$

		Maintenance costs per period (year)				
		1	2	3	4	5
Number of Trial	**Variable *z***	**MC (€)**	**MC (€)**	**MC (€)**	**MC (€)**	**MC (€)**
1	0.5%	136.4	408.9	478.4	864.5	692.9
2	1.6%	212.4	330.9	375.8	475.7	1062.4
3	1.6%	217.9	354.3	290.3	639.4	945.3
4	0.8%	210.7	274.3	455.0	765.2	835.5
5	4.2%	175.5	212.1	585.2	681.7	478.8
…						
10000	0.3%	112.7	319.8	718.2	520.5	938.2

Table 36 shows the output of the Monte Carlo simulation - the average and the standard deviation of z. The average of *z* is similar to the Risk-free rate (1.6%) but the standard deviation (volatility) of the variable in each period (30%) is different from the standard deviation (volatility) of the project.

Table 36 - Outputs of the Monte Carlo simulation

	z average	z s. deviation
Toyota Auris 1.6 – defender	1.5%	3.1%
Toyota Auris 2.0 – defender	1.6%	4.5%

The frequency of *z* and the graphical analysis of the Monte Carlo outputs are shown in figure 4 and 5.

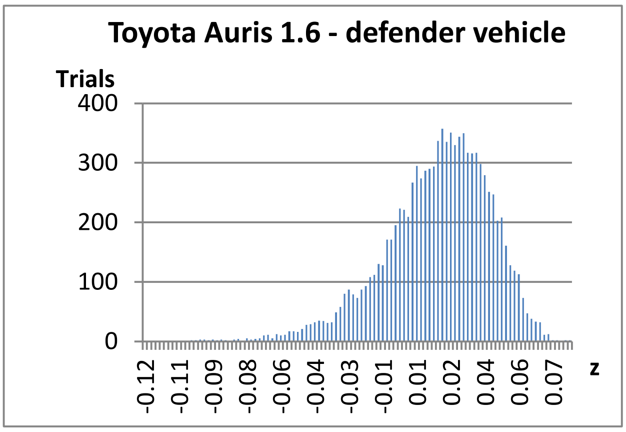

Figure 4 - Frequency of z for Toyota Auris 1.6

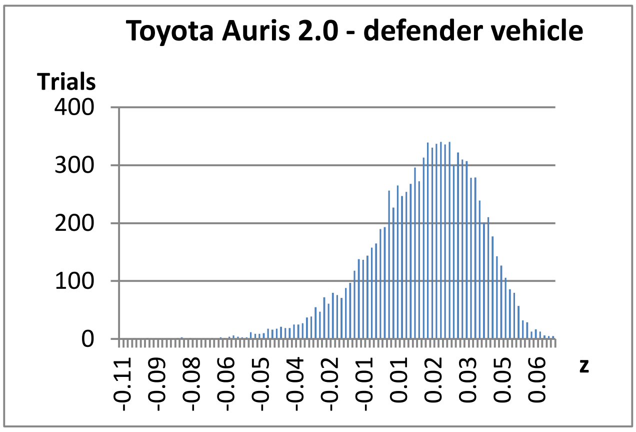

Figure 5 - Frequency of z for Toyota Auris 2.0

The first step to study replacement considering real options is to build the tree for the uncertainty variable – maintenance costs (see section 4.3.4). In order to do that we will need to determine the volatility of the maintenance costs.

The maintenance costs volatility that is going to be considered is the standard deviation of the maintenance costs concerning the time period (5 years). One of the outputs of the Monte Carlo simulation were 10,000 sets of values for the maintenance costs along the time period, this means that we will have 10,000 standard deviations (volatilities) for each set of trials. The value that this study will use for the maintenance costs volatility is the mean of these values (the mean of the volatility for each set of iterations). The Figures 6 and 7 show the different volatilities for each set of iterations.

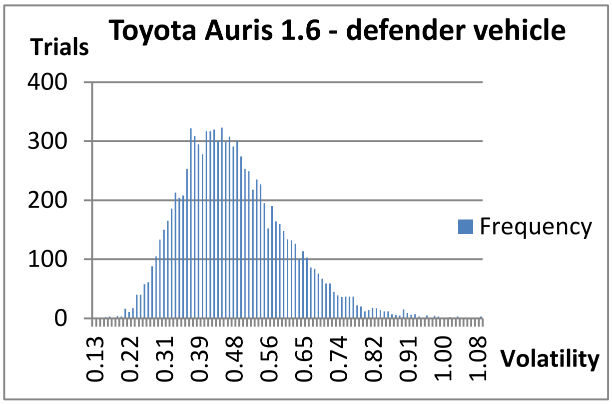

Figure 6 - Volatility per trial for the defender - Toyota Auris 1.6

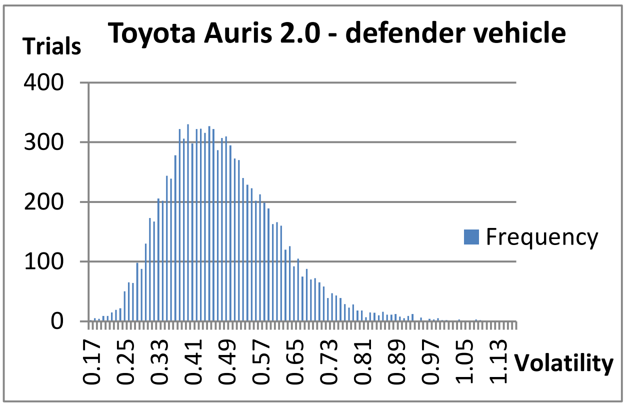

Figure 7 - Volatility per trial for the defender - Toyota Auris 2.0

With figures 6 and 7, we can verify that the volatility per trial has a central tendency. This means that the average value of the volatility is closer to the volatilities values that have an higher frequency. For that reason is suitable to use the mean as the volatility for the maintenance costs. For each defender vehicle (Toyota Auris 1.6 and Auris 2.0) the volatility is 0.48 - see figures 6 and 7. With the indicated volatility the "up movement" value is 1.62 and the "down movement" value is 0.62. Considering the values of the "up movement" and "down movement" and starting with the maintenance costs value in the first period (deterministic value equal to 163 € for the Toyota Auris 1.6 and equal to 216 € for the Toyota Auris 2.0), we can build the binomial tree for the maintenance costs - see tables 37 and 38.

Table 37 - MC binomial tree for Toyota Auris 1.6

Toyota Auris 1.6 (defender) - MC (€)				
Periods (years)				
1	2	3	4	5
				1115
			690	
		427		427
	264		264	
163		163		163
	101		101	
		63		63
			39	
				24

Table 38 - MC binomial tree for Toyota Auris 2.0

Toyota Auris 2.0 (defender) - MC (€)				
Periods (years)				
1	2	3	4	5
				1473
			911	
		564		564
	349		349	
216		216		216
	134		134	
		83		83
			51	
				32

The second step to study the replacement process considering real options is to build the tree for the FCFF (see section 4.3.4). Using deterministic values (taxes, insurance costs, inspection costs, depreciation costs and salvage value) and values estimated with the previous binomial tree (maintenance costs with uncertainty – table 37 and 38), each node was filled with the defender's FCFF (see tables 39 and 40). All of the values are negative because the investment value was not considered in this case. The values of the period 5 are the highest because the model considers the occurrence of the salvage value in this period (last period).

Table 39 - FCFF for Toyota Auris 1.6

Toyota Auris 1.6 (defender) - FCFF (€)				
Periods (years)				
1	2	3	4	5
				-7470
			-184	
		-416		-8013
	-545		-521	
-624		-624		-8222
	-674		-649	
		-704		-8301
			-699	
				-8332

Table 40 - FCFF for Toyota Auris 2.0

Toyota Auris 2.0 (defender) - FCFF (€)				
Periods (years)				
1	2	3	4	5
				-7345
			-207	
		-505		-8063
	-675		-651	
-780		-780		-8338
	-845		-821	
		-886		-8443
			-886	
				-8484

The third step is to calculate the accumulated value of the FCFF, recombining the previous binomial tree and starting from the nodes in the last period. Using tables 39 and 40, the process starts from the last node recombining the tree using the “up probabilities -p” and “down probabilities – (1-p)”. RNA uses “risk free” probabilities that adjust the cash-flows to the risk-free. These probabilities will be determined using the project volatility – 3.1% for Toyota Auris 1.6 and 4.5% for Toyota Auris 2.0. The value for *p* is 75% for Toyota Auris 1.6 and 67% for Toyota Auris 2.0. Tables 41 and 42 show the recombined binomial tree for each defender vehicle and the future FCFF values (see section 4.3.4).

Table 41 – Recombined future FCFF for Toyota Auris 1.6

Toyota Auris 1.6 (defender) - recombined FCFF (€)				
Periods (years)				
1	2	3	4	5
				-7470
			-7486	
		-7743		-8013
	-8244		-7938	
-8859		-8400		-8222
	-8964		-8112	
		-8652		-8301
			-8178	
				-8332

Table 42 - Recombined future FCFF for Toyota Auris 2.0

Toyota Auris 2.0 (defender) - recombined FCFF (€)				
Periods (years)				
1	2	3	4	5
				-7345
			-7465	
		-7881		-8063
	-8603		-8027	
-9378		-8667		-8338
	-9432		-8241	
		-8968		-8443
			-8324	
				-8484

The fourth step calculates the $EAC_{defender}$ with uncertainty. The values of the future cash-flows defined above (tables 41 and 42), were converted into an EAC in order to make possible the comparison among options with different maturities. In each period, the investment value related to the defender vehicle (residual value) was considered in order to compare the EAC of the challenger (deterministic value) with the defenders' EAC resulting from the binomial tree.

The fifth step calculates the value of the $EAC_{defender}$ with uncertainty minus the $EAC_{challenger}$ (deterministic value) – see tables 43 and 44. The yellow nodes are the positive values (nodes where the defender vehicle should be replaced because the defender annualized FCFF were higher than the challenger annualized FCFF). Terefore, the negative nodes ($EAC_{defender} < EAC_{challenger}$) correspond to the ones where the defender vehicle shouldn`t be replaced. The periods of tables 43 and 44 are related to the age of the vehicles - each period is related to the vehicles's age of the defender. At the first period, the defender vehicle has 4 more years to achieve its maturity. In what concern to the fifth period, it was not considered because in that period, the rent-a-car company is obliged (legal constraint) to replace the defender vehicle (in the fifth period the maturity was achieved).

Table 43 – EAC defender minus EAC challenger for Toyota Auris 1.6

Toyota Auris 1.6: $EAC_{defender} - EAC_{challenger}$ (€)			
Vehicle`s age (years)			
1	2	3	4
			274
		250	
	286		-186
386		-86	
	38		-362
		-215	
			-429

Table 44 – EAC defender minus EAC challenger for Toyota Auris 2.0

Toyota Auris 2.0: $EAC_{defender} - EAC_{challenger}$ (€)			
Vehicle`s age (years)			
1	2	3	4
			1072
		1084	
	1204		501
1453		681	
	918		283
		527	
			199

The binomial trees of Tables 43 and 44 show that the critical replacement timing occurs in the first period because in the indicated period, the $EAC_{defender}$ is always higher than the $EAC_{challenger}$. For Toyota Auris 1.6, in the third and fourth periods (last one) there are nodes with negative values (white cells) and positive values (yellow cells). This means that the uncertainty related to the maintenance costs produces replacement outputs that are not consistent and don´t produce unique solutions about the replacement problem in those periods.

The values between $EAC_{defender}$ and $EAC_{challenger}$ became closer along the time of replacement, meaning that there are serious doubts about the replacement decision. Thus, a light variation on the data can change the outcome of the model. Supporting this evidence, we can verify that in the first period, the $EAC_{defender} - EAC_{challenger}$ has the higher value of all the periods, signifying that an early decision will maximize the difference between the $EAC_{defender}$ and the $EAC_{challenger}$.

5.2.4 Replacement process without considering $C0_2$ emissions legal framework - real options approach

To analyze the replacement of the vehicles considering uncertainty related to the maintenance costs and excluding the effect of CO_2 emissions this section excludes legal framework related to CO_2 emissions - see section 2.1 and 2.2. The taxes IUC and ISV are function of two main components: engine size and CO_2 emissions. The components related to CO_2 emissions are not considered including the 40% discount of ISV (legal framework related to CO_2 emissions). The indicated assumption implies changes on the following variables: investment value, depreciation costs, ISV and IUC.

The project volatility excluding the effect of CO_2 emissions is 3.0% for Toyota Auris 1.6 and 4.3% for Toyota Auris 2.0. Tables 45 and 46 show $EAC_{defender}$ with uncertainty minus the $EAC_{challenger}$ (deterministic value) excluding the effect of CO_2 emissions. The yellow cells with the negative values represent the nodes where the defender vehicle should be replaced.

Table 45 – EAC defender minus EAC challenger for Toyota Auris 1.6

Toyota Auris 1.6: $EAC_{defender}$ – $EAC_{challenger}$ (€)			
Vehicle`s age (years)			
1	**2**	**3**	**4**
			50
		26	
	61		-413
160		-314	
	-190		-590
		-444	
			-658

Table 46 – EAC defender minus EAC challenger for Toyota Auris 2.0

Toyota Auris 2.0: $EAC_{defender}$ – $EAC_{challenger}$ (€)			
Vehicle`s age (years)			
1	**2**	**3**	**4**
			927
		938	
	1057		353
1304		532	
	767		133
		376	
			49

From the information included in tables 45 and 46, some conclusions can be taken. Thus, assuming uncertainty related to the maintenance costs and excluding the effect of CO_2 taxes and legal framework, the replacement of the defender vehicle for the challenger vehicle should occur in the first period (critical replacement timing). Comparing tables 43 and 44 (values considering CO_2 emissions legal framework) and tables 45 and 46 (values without considering CO_2 emissions legal framework), we can verify that the difference between $EAC_{defender}$ and $EAC_{challenger}$ is lower. The ISV reduction of 40% is the most relevant parameter for this difference. In what respect to Table 45, we can find nodes with negative values (white cells) and positive values (yellow cells) in the second, third and fourth periods, meaning that the binomial approach related to the maintenance costs with uncertainty doesn't produce unique solutions about the replacement problem in those periods.

6 Sensitivity Analysis

The goal of this section is to analyze the variation of EAC (deterministic approach) related with the replacement process considering different levels for the variables.

Another objective is to study the impact of the volatility related to maintenance costs on the replacement decision considering Real Options.

6.1 Sensitivity Analysis – Equivalent annual cost approach

As the standard variable, the section of the sensitivity analysis will use EAC considering CO_2 emissions (considering CO_2 legal framework) to assess the robustness of the models. The first variable analyzed is WACC. Therefore, Table 47 shows the EAC considering a minimum WACC of 2.1% and a maximum WACC of 3.3% - output from the Monte Carlo simulation for the cost of capital (see section 5.1.8). The percentage (%) in Table 47 is the variation of EAC comparing with the standard WACC of 2.9% employed in the study calculations.

Table 47 - EAC defender and challenger; xa: vehicle`s age of defender (years); %=variation of EAC; %=(EACx/EAC2.9%)/ EAC2.9% with x=2.1% or 3.3%

	Defenders								Challenger			
	Auris 1.6				Auris 2.0				Auris 1.8			
	(WACC=2.1%)		(WACC=3.3%)		(WACC=2.1%)		(WACC=3.3%)		(WACC=2.1%)		(WACC=3.3%)	
xa	**EAC (€)**	**%**	**EAC (€)**	**%**	**EAC (€)**	**%**	**EAC (€)**	**%**	**EAC (€)**	**%**	**EAC (€)**	**%**
1	3118	-3.8	3302.7	1.9	4291	-3.3	4510	1.7	2580.9	-4.5	2763.9	2.3
2	2971	-3.5	3132.1	1.8	4003	-3.0	4188	1.5				
3	2848	-3.2	2989.3	1.6	3768	-2.7	3925	1.4				
4	2730	-2.9	2852.3	1.5	3559	-2.4	3691	1.2				
5	-	-	-	-	-	-	-	-				

According the values of EAC, included in table 47, the replacement process produces the same critical timing of replacement (1 year) for all cases of even considering the perturbation of the WACC. However, there is a relevant difference in the critical replacement level - 2580.90€ (yellow cell) with a decrease of 4.5% for a WACC of 2.1% and 2763.90€ (green cell) representing an increase of 2.3% for a WACC of 3.3%. Thus, the challenger vehicle is more sensitive to the variation of WACC than the defender vehicles. However, this difference is not enough relevant to produce a change in the critical replacement timing, considering that the WACC varies from 2.1% to 3.3%. For occurring an update in the critical replacement timing, the WACC needed to produce that change would be very high. For instance, considering the defender Toyota Auris 1.6, the WACC needed to produce a change in the timing of replacement is 95%. In this case the best solution is to don`t replace the vehicle. This information seems to reveal that WACC may not be considered as a relevant variable to the decision of critical timing.

Knowing, the EAC analysis has a period of 5 years, it is also important to examine if different time horizons produce different results. Table 48 shows the results for a period of 4 years - the percentage (%) is the variation of EAC between a period of 4 years and a period of 5 years.

Table 48 - EAC and EAC variation for a time period of 4 years; xa: vehicle`s age of defender (years); %=variation of EAC; %=(EACx=4/EACx=5)/ EACx=5 where x is the number of periods

	Defenders				Challenger	
xa	**EAC (Auris 1.6) - €**	%	**EAC (Auris 2.0) - €**	%	**EAC (Auris 1.8) - €**	%
1	3376.29	4.2	4685.61	5.6	2665.07	-1.4
2	3206.05	4.2	4356.00	5.6		
3	3069.46	4.3	4090.90	5.6		
4	-	-	-	-		

Table 48 illustrates that a lower legal maturity (lower time period) decreases the $EAC_{challenger}$ and increases the $EAC_{defender}$. For the defender vehicles, the CRC (Capital recovery cost) increases and the EATC (Equivalent annualized total cost) tends to decrease but this difference is not enough to produce any change, because of that the $EAC_{defender}$ increases. For the challenger vehicle, in result of the perturbation, the CRC also increases but this increasing is offset by the EATC decreasing. Changing the time horizon to 4 years, the critical replacement timing is also 1 year and there is a reduction of 1.4% in the critical replacement level (yellow cell). Therefore, we can conclude that a lower time horizon benefits the replacement by the challenger vehicle.

Evolving in another direction, Table 49 displays the results for a time horizon of 6 years - the percentage (%) is the variation of EAC between a time horizon of 6 years and a standard time horizon of 5 years.

Table 49 - EAC and EAC variation for a time period of 6 years; xa: vehicle`s age of defender (years); %=(EACx=6/EACx=5)/ EACx=5 where x is the number of periods

	Defenders				Challenger	
xa	**EAC (Auris 1.6) - €**	%	**EAC (Auris 2.0) - €**	%	**EAC (Auris 1.8) - €**	%
1	3094.34	-4.5	4184.06	-5.7	2706.38	0.1
2	2923.74	-5.0	3870.85	-6.2		
3	2766.67	-6.0	3596.02	-7.1		
4	2596.65	-7.6	3258.39	-10.7		
5	2507.27	-	3164.88	-		
6	-	-	-	-		

Table 49 shows that a greater time period increases the $EAC_{challenger}$ (marginal increase) but decreases $EAC_{defender}$. For the defender vehicles the CRC decreases and the EATC tends to increase but this gain is not enough to produce any change, because of that the $EAC_{defender}$ decreases. For the challenger vehicle the CRC also decreases but this gain is offset by the EATC increase. The CRC decreasing is similar to the EATC increasing and for that reason the EAC increasing for the challenger vehicle is only 0.1%. For a time period of 6 years the critical replacement timing is also 1 year and there is an increase of 0.1% in the critical replacement level (yellow cell). Considering as defender the vehicle Toyota Auris 1.6 (with a period of 6 years) if the decision about replacement is delay until the fourth period the best decision is to don`t replace de fleet. We can conclude that a greater time period benefits the defender vehicles.

Table 50 shows the EAC variation considering the declining balance method (see chapter 4.1.3) - the percentage (%) is the variation of EAC between the declining balance method and the linear method.

Table 50 - EAC and EAC variation level considering declining balance method (depreciation); %=(EAC declining balance method/EAC linear method)/ EAC linear method

	Defenders				Challenger	
xa	**EAC (Auris 1.6) - €**	%	**EAC (Auris 2.0) - €**	%	**EAC (Auris 1.8) - €**	%
1	3221.55	-0.6	4412.11	-0.6		
2	3057.31	-0.7	4099.72	-0.6		
3	2926.59	-0.5	3852.18	-0.5	2689.89	-0.6
4	2811.45	0.0	3647.22	0.0		
5	-	-	-	-		

The critical level of replacement reduces 0.6% - yellow cell. The EAC of both vehicles (defenders and challenger) tends to decrease. The replacement timing is also 1 year. This outcome is similar to the outcome obtained with the linear depreciation method. Thus, in this context, the depreciation method doesn’t constitute a relevant factor for the replacement process but It´s possible to conclude that a faster depreciation produces lower EAC values.

Table 51 presents the EAC results without considering the 40% ISV reduction for hybrid vehicles - the percentage (%) is the variation of EAC between an EAC without the 40% reduction and an EAC with the 40% reduction

Table 51 - EAC defender and challenger without considering the 40% ISV reduction for hybrid vehicles; xa: vehicle`s age of defender (years); %=($EAC_{without\ 40\%\ reduction}$ /$EAC_{with\ 40\%\ reduction}$)/ $EAC_{with\ 40\%\ reduction}$

	Defenders				Challenger	
xa	**EAC (Auris 1.6) - €**	%	**EAC (Auris 2.0) - €**	%	**EAC (Auris 1.8) - €**	%
1	3241.08	0.0	4437.04	0.0		
2	3078.26	0.0	4126.46	0.0		
3	2942.30	0.0	3872.23	0.0	2917.27	8.0
4	2811.45	0.0	3647.22	0.0		
5	-	-	-	-		

The $EAC_{defenders}$ is the same (0% variation) because the challenger vehicle is the only hybrid vehicle. With the previous table we can conclude that without the 40% ISV reduction for hybrid vehicles the critical replacement timing is also in the first period but there is an increase of 8.0% in the critical replacement level (yellow cell). This increase is relevant to the replacement process because in the fourth period the $EAC_{defender} < EAC_{challenger}$ considering the defender Toyota Auris 1.6.

The following tables (tables 52, 53 and 54) compare the $EAC_{defender}$ of Toyota Auris 1.6 with $EAC_{challenger}$ of Toyota Auris 1.8. Table 52 presents the critical replacement level variation considering different levels of maintenance costs and salvage value in the last period (vehicles`age = 5) for the challenger vehicle. Table 52 considers a standard critical replacement level of 2702.41 € (see table 29 – critical replacement level considering CO_2 legal framework) to determine the variations of the replacement level (%). The yellow cells (table 52) represent the variation of the critical replacement level that is related to the replacement of the defender vehicle by the challenger vehicle. The white cells (table 52) represent the variation of the critical replacement level that is related to mantain the defender vehicle until the last year.

Table 52 - Critical replacement level variation considering different levels of maintenance costs and salvage value in the last period (vehicles` age = 5) for the challenger vehicle; %=( Critical Replacement Level $_{\text{with SV or MC variation}}$ – 2702.41)/2702.41; yellow cell: replace; white cell: don` t replace

	Challenger - Auris 1.8 Replacement Level %	Maintenance costs %				
		0%	**10%**	**20%**	**30%**	**40%**
Salvage value % in the last period	**-40%**	19.9%	19.9%	19.9%	19.9%	19.9%
	-30%	19.9%	19.9%	19.9%	19.9%	19.9%
	-20%	12.0%	13.4%	14.8%	16.3%	17.7%
	-10%	6.0%	7.4%	8.9%	10.3%	11.7%
	0%	0.0%	1.4%	2.9%	4.3%	5.7%

The value 19.9% appears in the two first lines (white cells) because the variable variations produce an $EAC_{challenger}$ higher than the $EAC_{defender}$ in each period, this means that the critical replacement level is always equal to the $EAC_{defender}$ in the first period (vehicles` age = 1) – the EAC to maintain the defender fleet until the last year - the best solution is to don` t replace the defender vehicle. With table 52, we can conclude that the replacement process produces different critical replacement timings when the salvage value variation of the challenger vehicle is higher than -20%. Table 53 presents the critical replacement level variation considering different levels of maintenance costs and investment level for the challenger vehicle. Table 53 also considers a standard critical replacement level of 2702.41€ (see table 29 – critical replacement level considering CO_2 legal framework).

Table 53 - Critical replacement level variation considering different levels of maintenance costs and investment level for the challenger vehicle; %=(Critical Replacement Level $_{\text{with I or MC variation}}$ – 2702.41)/2702.41; yellow cell: replace; white cell: don` t replace

Invest. Value %	Challenger - Auris 1.8 Replacement Level %I	Maintenance costs % 0%	10%	20%	30%	40%
	40%	19.9%	19.9%	19.9%	19.9%	19.9%
	30%	19.9%	19.9%	19.9%	19.9%	19.9%
	20%	19.9%	19.9%	19.9%	19.9%	19.9%
	10%	13.3%	14.7%	16.2%	17.6%	19.0%
	0%	0.0%	1.4%	2.9%	4.3%	5.7%

With table 53, we can conclude that the replacement process produces different critical replacement timings when the investment value variation of the challenger vehicle is higher than 10%. Table 54 presents the critical replacement level variation considering different levels of maintenance costs and salvage value in the last period (vehicles`age = 5) for the defender vehicle Toyota Auris 1.6. Table 53 also considers a standard critical replacement level of 2702.41€ (see table 29 – critical replacement level considering CO_2 legal framework).

Table 54 - Critical replacement level variation considering different levels of maintenance costs and salvage value in the last period (vehicles` age = 5) for the defender vehicle Toyota Auris 1.6; %=( Critical Replacement Level $_{\text{with I or MC variation}}$ – 2702.41)/2702.41; yellow cell: replace; white cell: don` t replace

Salvage value % in the last period	Defender - Auris 1.6 Replacement Level %	Maintenance costs % 0%	-10%	-20%	-30%	-40%
	40%	-10.8%	-12.4%	-14.1%	-15.7%	-17.4%
	30%	-3.1%	-4.8%	-6.4%	-8.1%	-9.7%
	20%	0.0%	0.0%	0.0%	-0.1%	-2.0%
	10%	0.0%	0.0%	0.0%	0.0%	0.0%
	0%	0.0%	0.0%	0.0%	0.0%	0.0%

The value 0.0% appears in the two last lines and in the beginning of the third line (yellow cells) because the variable variations produce an $EAC_{challenger}$ lower than the $EAC_{defender}$, this means that the critical replacement level is always equal to 2702.41€ (critical replacement level with CO_2 legal framework). With table 54, we can conclude that the replacement process produces different critical replacement timings when the salvage value variation is higher than 20% or the salvage value variation is equal to 20% and the maintenance costs variation is higher than 20% considering the defender vehicle Toyota Auris 1.6.

6.2 Sensitivity Analysis – Real Options approach

The sensitivity analysis to the real options approach will use values considering CO_2 emissions (considering real options values with CO_2 legal framework – see section 5.1.9). For both defender vehicles (Toyota Auris 1.6 and Auris 2.0), the volatility for the maintenance costs is 0.48. This value was determined using an average value (see figures 6 and 7 – section 5.2.3) with a standard deviation of 0.14 (for both defender vehicles). The sensitivity analysis will perturb the value of the volatility among an interval ranging from 0.34 and 0.62 (considering an interval related to one standard deviation) to evaluate the effect of the variation of the maintenance costs volatility in the replacement process. The indicated interval (0.34-0.62) includes 74% of the volatility values, generated by the Monte Carlo simulation for Toyota Auris 1.6 and 76% of the volatility values generated by the Monte Carlo simulation for Toyota Auris 2.0.

Tables 55 and 56 show the binomial tree of the replacement option given by the $EAC_{defender}$ with uncertainty minus the $EAC_{challenger}$ (constant and deterministic value) considering a volatility of the maintenance costs equal to 0.34 (lower boundary of the interval). The yellow cells represent the nodes where the $EAC_{defender} > EAC_{challenger}$.

Table 55 – EAC defender minus EAC challenger for Toyota Auris 1.6

Toyota Auris 1.6: $EAC_{defender} - EAC_{challenger}$ (€)			
Vehicle`s age (years)			
1	2	3	4
			-30
		55	
	163		-247
311		-117	
	26		-358
		-205	
			-414

Table 56 – EAC defender minus EAC challenger for Toyota Auris 2.0

Toyota Auris 2.0: $EAC_{defender} - EAC_{challenger}$ (€)			
Vehicle`s age (years)			
1	2	3	4
			703
		861	
	1074		429
1373		649	
	910		290
		542	
			220

Comparing the values contained in tables 55 and 56 with tables 43 and 44 (section 5.2.3), we can state that a lower volatility for the maintenance costs produces a lower $EAC_{defender}$ minus $EAC_{challenger}$ value but the replacement timing remains the same (first period). For Toyota Auris 2.0 (Table 56) the nodes are all positives, meaning that in each node and period the replacement decision is to replace the defender vehicle Toyota Auris 2.0. For Toyota Auris 1.6 (Table 55) in the first and second periods the nodes are all positives, meaning that the replacement decision is to replace the defender vehicle Toyota Auris 1.6. In the third period there are nodes with negative values (white cells) and positive values (yellow cells). This means that the uncertainty related to the maintenance costs produces replacement outputs that are not consistent and for that reason there are doubts about the adequate replacement decision. In the last period the tree nodes are all negative (white cells). This means that if the replacement decision is delay until the fourth period the decision is to don`t replace the defender vehicle.

Tables 57 and 58 show the binomial tree of $EAC_{defender}$ with uncertainty minus the $EAC_{challenger}$ (deterministic value) considering a volatility of the maintenance costs equal to 0.62 (maximum value of the interval). The yellow cells represent the nodes where the $EAC_{defender}$ > $EAC_{challenge}$ and for that reason the defender vehicle should be replaced.

Table 57 – EAC defender minus EAC challenger for Toyota Auris 1.6

Toyota Auris 1.6: $EAC_{defender}$ – $EAC_{challenger}$ (€)			
Vehicle`s age (years)			
1	**2**	**3**	**4**
			797
		567	
	477		-104
502		-46	
	56		-365
		-223	
			-440

Table 58 – EAC defender minus EAC challenger for Toyota Auris 2.0

Toyota Auris 2.0: $EAC_{defender}$ – $EAC_{challenger}$ (€)			
Vehicle`s age (years)			
1	**2**	**3**	**4**
			1702
		1441	
	1403		597
1574		724	
	933		278
		516	
			185

The comparison between tables 57 and 58 with tables 43 and 44 (section 5.2.3) allow to understand that higher volatilities of the maintenance costs produce higher replacement values and option values ($EAC_{defender}$ minus $EAC_{challenger}$). However, the critical replacement timing remains the same (first period).

7 Conclusions

This document shows the possibility to define suitable models to evaluate replacement timing and level of vehicles from the firm`s point of view (rent-a-car). Considering the existing legal framework related to CO_2 emissions, the models consider the environmental impacts in the replacement process. In that way, the document defined two models of analysis (Equivalent Annual Cost and Real Options) in order to study the replacement problem considering two different environments: a deterministic environment and a stochastic (uncertain) environment. The data used to fill the models of this study came from different sources (Toyota, Guia do Automóvel, Império Bonança, legal framework – CISV and CIUC). The case study was composed using market data with the aim to perform a formal evaluation in order to create a set of outputs from different methodologies.

The main goal of this study was to define an economic model to analyze non-identical replacements of vehicles belonging to a rent-a-car company. Another relevant goal was to understand the impact of CO_2 emissions on the replacement timing and replacement level. This study concluded that legal framework related to CO_2 emissions benefit the hybrid vehicle (challenger vehicle).

According this study, the ISV tax reduction produces a motivation for replacement and the ISV tax reduction of 40% analyzed in this study supports that conclusion. According Portuguese law the ISV reduction happens for electric, hybrid vehicles and vehicles with $C0_2$ emissions lower than 120g/km. This study also shows that the taxes related to CO_2 emissions and the legal framework related to CO_2 emissions are more relevant to the replacement timing if the decision about replacement is delay after the first year.

The output of the models that were created shows that there is the tendency to have a fast replacement. The study shows that the tendency is to replace the vehicle in the first period (first year). Rent-a-car companies also perform fast replacements for their fleets. Two of the biggest rent-a-car companies operating in Portugal - Avis e Europcar - perform average replacement timings of 6 months to 1 year (Europcar Portugal, 2015; Avis Portugal, 2015)

The Capital Recovery Cost (CRC) is the most important component because EAC is more sensitive to CRC variations. For that reason, the investment value, the ISV and the residual value are very important and relevant to the replacement decision. Slightly variations of the investment value and the salvage value can produce different decisions about the replacement process.

The maturity of the vehicles is also relevant to the outputs of the model. The higher maturity of the challenger vehicle and the lower maturity of the defender vehicle produce higher outputs (EAC) for the defender vehicle and lower outputs (EAC) for the challenger vehicle. This means that the maturity effect benefits the challenger vehicle and a faster replacement. Vehicles with higher maturities tend to produce lower EACs comparing with vehicles with lower maturities. This means that the legal maturity for a rent-a-car vehicle can be relevant to the decision

Another conclusion that it`s possible to extract from the models that were used is that if we delay the decision about replacement the values between $EAC_{defender}$ and $EAC_{challenger}$ became closer in some cases and for a specific data. This means that if we decide to wait the tendency is to have more doubt about the decision and a lightly variation of the data can change the outcome of the model.

The Real Options model produced similar results comparing with the EAC model. The Real Options model was important to analyze the case study considering uncertainty related to the maintenance costs and to verify the consistence of the results produced by the deterministic model (EAC). The MC is a variable that is not relevant to the decision of critical timing but it is relevant if the decision about replacement is delay after the first period, because of that the MC is considered as an important variable in the replacement process.

The depreciation cost is also very important. A cost structure with higher depreciation costs tends to have lower EACs. A faster depreciation produces faster replacements and this information can be used to benefit the vehicle that has lower CO_2 emissions. Faster depreciations for vehicles with lower CO_2 emissions and slower depreciations for vehicles with higher CO_2 emissions can produce a positive impact in the replacement decisions.

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Annex 1 - Vehicles sold in Portugal and vehicles sold in Portugal to rent-a-car companies (ARAC, 2015)

Table 59 - Vehicles sold in Portugal (ACAP) and vehicles sold in Portugal to rent-a-car companies

	ARAC		***Var% Acumul 11/10***	**ACAP**		***Var% Acumul 11/10***
BRANDS	**2011**	2010		**2011**	2010	
ALFA ROMEO	**99**	18	**450.00%**	**1 851**	2.275	**-18.64%**
ASIA MOTORS	**0**	0	**0.00%**	**ND**	ND	**ND**
ASTON MARTIN	**0**	0	**0.00%**	**12**	21	**-42.86%**
AUDI	**896**	631	**42.00%**	**6 622**	8 403	**-21.19%**
BENTLEY	**0**	0	**0.00%**	**4**	4	**0.00%**
BERTONE	**0**	0	**0.00%**	**ND**	ND	**ND**
BMW	**404**	308	**31.17%**	**7 356**	9 714	**-24.27%**
CADILAC	**0**	0	**0.00%**	**ND**	ND	**ND**
CATHERHAM	**0**	0	**0.00%**	**ND**	ND	**ND**
CHEVROLET	**444**	726	**-38.84%**	**3 205**	6 359	**-49.60%**
CHRYSLER	**0**	0	**0.00%**	**22**	84	**-73.81%**
CITRÖEN	**1 485**	1 540	**-3.57%**	**9 090**	13 369	**-32.01%**
CORVETE	**0**	0	**0.00%**	**ND**	ND	**ND**
DACIA	**0**	0	**0.00%**	**2 231**	1 658	**34.56%**
DAIHATSU	**0**	0	**0.00%**	**0**	56	**-100.00%**
DODGE	**0**	0	**0.00%**	**110**	492	**-77.64%**
FERRARI	**0**	0	**0.00%**	**18**	23	**-21.74%**
FIAT	**1 970**	2 438	**-19.20%**	**6 996**	10 657	**-34.35%**
FORD	**2 607**	2 909	**-10.38%**	**10 616**	15 386	**-31.00%**
GALLOPER	**0**	0	**0.00%**	**ND**	ND	**ND**
HONDA	**65**	29	**124.14%**	**1 827**	3 775	**-51.60%**
HYUNDAI	**551**	611	**-9.82%**	**2 782**	3 152	**-11.74%**
INNOCENTI	**0**	0	**0.00%**	**ND**	ND	**ND**
JAGUAR	**0**	0	**0.00%**	**132**	258	**-48.84%**
JEEP	**0**	0	**0.00%**	**9**	0	**100.00%**
KIA	**488**	532	**-8.27%**	**2.701**	3.889	**-30.55%**
LADA	**0**	0	**0,00%**	**ND**	ND	**ND**

LAMBORGHINI	**0**	0	**0,00%**	**3**	6	**-50,00%**
LANCIA	**51**	14	**264.29%**	**388**	637	**-39.09%**
LAND ROVER	**0**	0	**0.00%**	**191**	104	**83.65%**
LEXUS	**4**	1	**300.00%**	**273**	286	**-4.55%**
LOTUS	**0**	0	**0.00%**	**1**	1	**0.00%**
MASERATI	**0**	0	**0.00%**	**1**	4	**-75.00%**
MAZDA	**97**	486	**-80.04%**	**1 033**	3 273	**-68.44%**
MERCEDES	**768**	565	**35.93%**	**7 095**	9 020	**-21.34%**
MG	**0**	0	**0.00%**	**ND**	ND	**ND**
MINI	**79**	12	**558.33%**	**1 639**	1 631	**0.49%**
MITSUBISHI	**483**	845	**-42.84%**	**2 283**	3 416	**-33.17%**
MORGAN	**0**	0	**0.00%**	**ND**	ND	**ND**
NISSAN	**954**	983	**-2.95%**	**6 654**	8 311	**-19.94%**
OPEL	**3 264**	3 624	**-9.93%**	**11 372**	17 257	**-34.10%**
PEUGEOT	**3 039**	3 915	**-22.38%**	**12 870**	18 048	**-28.69%**
PONTIAC	**0**	0	**0.00%**	**ND**	ND	**ND**
PORSCHE	**4**	0	**100.00%**	**258**	468	**-44.87%**
RENAULT	**3 705**	4 075	**-9.08%**	**16 340**	26 197	**-37.63%**
ROLLS-ROYCE	**0**	0	**0.00%**	**ND**	ND	**ND**
ROVER	**0**	0	**0.00%**	**ND**	ND	**ND**
SAAB	**159**	0	**100.00%**	**114**	31	**267.74%**
SEAT	**2 592**	2 551	**1.61%**	**8 436**	13 062	**-35.42%**
SKODA	**297**	485	**-38.76%**	**2 711**	4 540	**-40.29%**
SMART	**195**	91	**114.29%**	**2 229**	2 544	**-12.38%**
SSANGYONG	**0**	0	**0.00%**	**0**	1	**-100.00%**
SUBARU	**0**	0	**0.00%**	**1**	60	**-98.33%**
SUZUKI	**21**	27	**-22.22%**	**566**	1 235	**-54.17%**
TATA	**0**	0	**0.00%**	**ND**	ND	**ND**
TOYOTA	**909**	1 421	**-36.03%**	**5 519**	11 499	**-52.00%**
TVR	**0**	0	**0,00%**	**ND**	ND	**ND**
VOLKSWAGEN	**4 254**	3 365	**26.42%**	**14 912**	18 814	**-20.74%**
VOLVO	**148**	129	**14.73%**	**2 869**	3 443	**-16.67%**
TOTAL	**30 032**	**32 331**	**-7 11%**	**153 342**	**223 463**	**-31.38%**

Annex 2 - Beta calculation for Avis, Hertz and Europcar Companies – data from (Investing.com)

- ## Avis

Table 60 - Beta calculation for Avis

Levered Beta	1.27*[1]
Unlevered Beta	0.07
Debt (millions €)	16304
Equity (millions €)	665
tax rate	35%*[2]

*1 – Considering one year of daily stock prices (7th Oct. 2014 until 7th Oct. 2015).

*2 – Standard corporate income tax rate of USA (Corporate Tax rates, 2015).

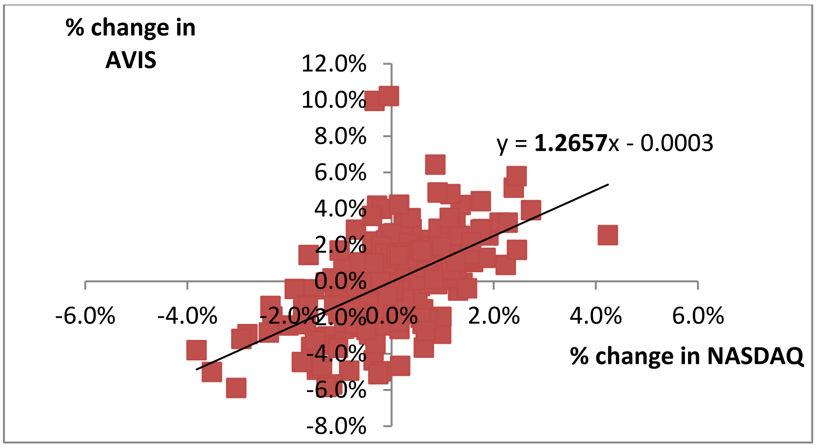

Figure 8 - Beta graphical analysis for Avis

- ## Hertz

Table 61 - Beta calculation for Hertz

Levered Beta	1.72*[1]
Unlevered Beta	0.26
Debt (millions €)	21521
Equity (millions €)	2464
tax rate	35%*[2]

*1 – Considering one year of daily stock prices (7th Oct. 2014 until 7th Oct. 2015).

*2 – Standard corporate income tax rate of USA (Corporate Tax rates, 2015)

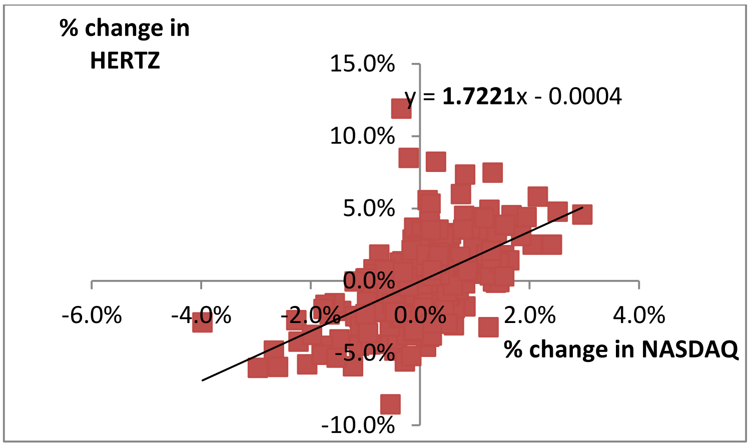

Figure 9 - Beta graphical analysis for Hertz

- ## Europcar

Table 62 - Beta calculation for Europcar

Levered Beta	0.47*[3]
Unlevered Beta	0.03
Debt (millions €)	3789.26
Equity (millions €)	157.19
tax rate	33.33%*[4]

*3 – Considering three months of daily stock prices (the company is publicly traded on Paris stock market since 26th June).

*4 – Standard corporate income tax rate of France (Corporate Tax rates, 2015).

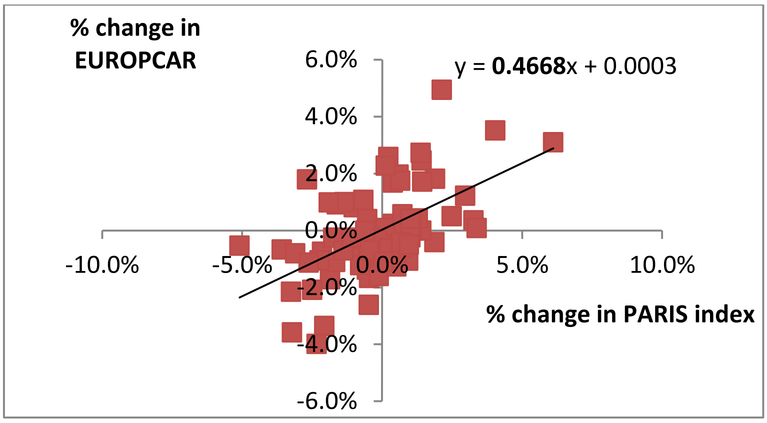

Figure 10 - Beta graphical analysis for Europcar

Annex 3 - Debt, Equity, Net income and number of companies for the rent-a-car portuguese market (INE, 2015)

Table 63 - Number of rent-a-car companies in the Portuguese market

Period	Number of companies
2014	476
2013	459
2012	465
2011	478
2010	465
2009	469
2008	479
2007	459
2006	448

Table 64 - Values for Debt, Equity and Net income of the rent-a-car Portuguese market. Source: INE

Period (years)	Net income		Total Capital		Equity		Debt	
	€		€		€		€	
2014		X	2497608417	//	428784980	//	2068823437	//
2013	17786696		2432524076		302922886		2129601190	
2012	-54734349		2874230340		246073864		2628156476	
2011	-51481620		3138711547		280895877		2857815670	
2010	⊥:		⊥:		⊥:		⊥:	
2009	-3111639		3122216123		298584831		2823631292	
2008	-13044205		3338379967		304408790		3033971177	
2007	17227367		3028193823		313130937		2715062886	
2006	12852828		2844350505		280339292		2564011213	
2005	-673722		2674930937		258215836		2416715101	
2004	-1030016		2227412984		213238862		2014174122	

//: Preliminar data
⊥: Series break

Annex 4 - Interests and obtained loans for the rent-a-car Portuguese market (INE, 2015)

Table 65 - Interests and obtained loans for the rent-a-car Portuguese market

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Period (year)	Interest payable and similar expenses	Obtained loans	
		(non-current liabilities)	(current liabilities)
	€		
2010	100308629	1286630502	694117065
2011	93901484	1313325791	841988156
2012	91517459	1114662714	749123,030
2013	74968720	822139123	543238830